# A Lunar Radar Navigation Concept

[Unclassified Title]

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## ABSTRACT

A navigation system concept is described that utilizes the moon as a reflector in a bistatic radar system. By measuring the range and range rate of the receiver relative to the moon, the location of the receiver on the earth's surface can be determined in both longitude and latitude. Lunar radar observations have shown that lunar range and range rate measurement accuracies equivalent to ±30 m can be achieved. By placing a transponder on the moon, the basic measurement accuracy could be improved by a factor of 5. However, for operational measurements where rapid readout is required, these accuracies would probably be degraded by a factor of 5 to 10. Placing three transmitters at appropriate locations on the earth's surface will provide worldwide coverage. A transmitter with an average power of 2 MW and a transmitting aperture of 170 m would supply a S/N ratio sufficient for reliable position determination with a dipole antenna receiver. One possible radar receiver configuration incorporating both a search and track mode is given.

A mathematical analysis of the coverage and the effective position accuracy indicates that (a) worldwide coverage is available, (b) time coverage is restricted to 50 percent on the average, but the time distribution of the coverage varies over a monthly period, (c) at low latitudes, the effective location accuracy is a function of the moon's declination, and (d) the optimum accuracy is obtained at high latitudes.

While restricted to some extent in coverage as described above, and more sensitive to systematic errors in the ship's velocity, the lunar radar navigation system, when compared with other worldwide radio navigation systems such as Omega and Transit, can achieve higher accuracies, is less vulnerable to jamming, and can also provide an independent, one-way communication channel. With improved technological developments and active reflectors on the moon, position accuracies of the order of several meters appear possible.

## PROBLEM STATUS

This is a final report on one phase of the problem, work is continuing on the project.

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# A LUNAR RADAR NAVIGATION CONCEPT [Unclassified Title]

## INTRODUCTION

Many different methods have been employed to determine the position of ships on the ocean or of other unknown locations on the earth's surface. To achieve the requirements of increasing accuracy, and all-weather, global, and 24-hour coverage, several radio navigation methods have been developed. Among these, the Transit satellites and the Omega system have been the most promising under development. However, practical considerations have necessitated some compromise between the various requirements. In particular, propagation uncertainties in Omega and orbit uncertainties of the Transit satellites limit the potential accuracy that can be achieved.

The lunar navigation system to be described in this report can overcome some of the limitations of the other systems and should improve the potential accuracy by an order of magnitude. This is achieved primarily by measuring the position of the moon simultaneously with the position of the unknown location. To obtain optimum accuracies ( $\pm 5$  m), a lunar transponder may be required. In addition, the system is less vulnerable to jamming and provides one-way communication between the master station and the unknown location. The basic disadvantage of the system is that the moon is above the horizon for only about 50 percent of the time.

## BASIC CONCEPT OF THE LUNAR NAVIGATION SYSTEM

The lunar navigation system utilizes the moon as the reflector in a bistatic radar system where the transmitter is at a known location and the receiver determines its position on the earth's surface from measurements of its distance and velocity relative to the moon. The transmitting station measures simultaneously its own distance and velocity relative to the moon and corrects the transmitted signal for variations of the lunar topography and the lunar ephemeris errors. The receiving station derives its position by applying the known lunar ephemeris to the range and velocity measurements. The two measurements provide the necessary corrections to both longitude and latitude of the unknown location. To provide worldwide coverage with some overlap, three ground-based transmitting stations are required, separated by 120° in longitude and placed at moderate latitudes. The system is worldwide except for the limitations imposed by the timevarying geometrical configuration of the earth-moon system, which is discussed in the next two sections.

## BASIC LIMITATIONS OF COVERAGE

There is a basic limitation of coverage due to the availability of the moon at a given time for a given location. The moon is available at any given location only when it is above the horizon. The altitude of the moon (h) at a given location is given by

$$\sin h = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos \gamma , \qquad (1)$$

where

- $\phi$  is the geocentric latitude of the receiver,
- $\delta$  is the declination of the moon, and
- $\gamma$  is the local hour angle.

The limits of visibility of the moon are given by Eq. (1) for h = 0. This yields the usable local hour angle for a given latitude and lunar declination as expressed in Eq. (2):

$$\cos \gamma = -\tan \varphi \tan \delta$$
. (2)

In Fig. 1, the usable hour angle per day is plotted as a function of latitude and different declinations of the moon. For a declination of 0°, the limiting hour angle is  $\pm 90^{\circ}$  or 12 hours for all latitudes. For positive declination, the available observation time per day increases with latitude until 24-hour coverage is obtained at high latitudes. At large negative latitudes for the same day the available observation time is reduced. The opposite behavior as a function of latitude occurs for negative declination. As the moon changes its declination from -28° to +28° in two weeks, the moon at latitudes close to the poles will be observable for at least two out of every four weeks.

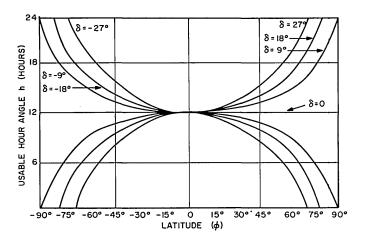
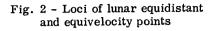
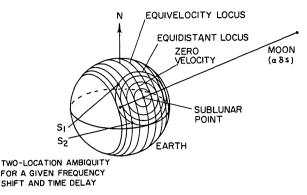


Fig. 1 - Visibility of the moon vs latitude and the moon's declination

## ERROR ANALYSIS

The locus of all points on the earth's surface which are equidistant from the moon at a given time is a circle on the earth's globe. This circle is located in a plane normal to the sublunar distance with the center of the circle positioned below the sublunar point. (See Fig. 2.) As the earth rotates around its axis, each point on the earth's surface will either approach or recede from the moon, except for the stationary points on the great circle which is located in a plane passing through the sublunar point and the earth's axis of rotation. The loci of equivelocity points are small circles on the globe, parallel to the plane containing the great circle of zero velocity. The range and range rate measured





by the observer will establish an equidistance and an equivelocity circle. The intersection of these two circles determines the location of the observer.

In general the intersection of the equidistant and equivelocity circles corresponding to one measurement will provide two possible locations on the earth's surface (Fig. 2). However, this ambiguity can be removed, since the approximate location of the measured position is generally known. The angle of intersection of the two circles determines the effective accuracy with which the two required quantities (longitude and latitude) can be determined. In general, the accuracy of the longitude is less dependent than that of the latitude on the angle of intersection of the two circles. The latitude error increases as the two circles approach each other tangentially. Since the sublunar point on the earth's surface changes with both a daily and a monthly period, the relative position of the two circles will change with time, and thus the effective error of the longitude and the latitude will be a function of both time and position.

In Appendix A, the effective error for both longitude and latitude has been derived. In the computations it has been assumed that the measurement errors of time delay and frequency are equal for the most efficient operation, and that the corrections for errors in the lunar ephemeris and for variations of the lunar topography have been made by the transmitting station. The effective errors are given by Eqs. (3) and (4):

$$\sigma_{\Lambda x} = \frac{\sqrt{\sin^2 \varphi + \cos^2 \varphi \, \tan^2 \delta - \sin \, 2\varphi \, \tan \, \delta \, \cos \, \gamma}}{\sin \, \varphi - \cos \, \varphi \, \tan \, \delta \, \cos \, \gamma} \, \sigma_m \, , \tag{3}$$

$$\sigma_{\Lambda y} = \frac{1}{\sin \varphi - \cos \varphi \tan \delta \cos \gamma} \sigma_m , \qquad (4)$$

where  $\sigma_{\Lambda x}$ ,  $\sigma_{\Lambda y}$ , and  $\sigma_m$  are the standard deviations in linear dimensions (meters) of longitude, latitude, and of the measurements, respectively.

These expressions give the effective errors of  $\Delta x$  and  $\Delta y$  as a function of latitude  $\varphi$ , the local hour angle y, and the moon's declination  $\delta$ . For some limiting cases these errors can be evaluated in a simple manner. For example, for  $\delta$  = 0 and  $\sigma_m$  = 1 m,

$$\sigma_{\Delta x} = 1 m$$
,  $\sigma_{\Delta y} = \frac{1}{\sin \varphi} m$ .

The effective error of the longitude is constant and equals the measurement accuracy at all locations. However, the error in the latitude increases as the latitude approaches zero (equator). Similarly, if  $\varphi = 0$  and  $\delta \neq 0$ ,

$$\sigma_{\Delta x} = \frac{1}{\cos \gamma} m$$
,  $\sigma_{\Delta y} = \frac{1}{\tan \delta \cos \gamma} m$ .

In this case, the errors of both longitude and latitude are a function of the local hour angle and are minimum at transit.

Data for a complete coverage diagram based on Eqs. (3) and (4) and the visibility of the moon discussed previously are derived in Appendix A and given in Tables A1 through A7. In general, the effective accuracies are best at high latitudes.

#### MEASUREMENT ERRORS

The effects of the ionosphere and the atmosphere on the time delay and frequency are negligible at microwave frequencies for measurement errors of the order of 100 m. For more accurate measurements, corrections for typical atmospheric conditions can reduce errors by an order of magnitude. This may become necessary if the full measurement accuracy of the lunar transponder is to be utilized.

The main modification of the radar signal is caused by the complex reflection characteristics of the moon. The varying topography will spread the leading edge by extending the buildup of the pulse anywhere between 5 and 20 microseconds (Fig. 3 shows a typical radar echo). In addition, the frequency is spread by the apparent rotation of the moon by about one part in  $10^8$ . However, by measuring the frequency spread in the first intervals of range (about 10 to  $100~\mu \rm sec$ ), the spread is reduced by an order of magnitude. The measurement consists of finding and establishing the centroid of both the leading edge and the frequency spectrum with an accuracy corresponding to  $1~\mu \rm sec$  in time and a frequency shift of one part in  $10^{10}$ . This is probably best accomplished by utilizing a tracking gate in range and a direct frequency count to establish the mean time delay and frequency of the received signal.

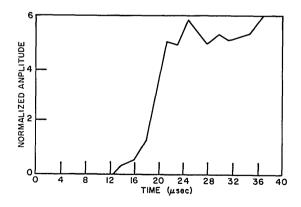


Fig. 3 - Lunar radar echo of May 15, 1968 (Observation 187, No. 75), integration time 0.1 sec

Measurements of the lunar distance with the NRL lunar radar system, which employs a basic 1.2- $\mu$ sec range resolution, have shown that the internal consistency can be maintained to within  $\pm 0.1~\mu$ sec or  $\pm 30~m$  in flat areas. The range variations for a few adjacent areas are given in Table 1. However, these measurements are obtained from the first point of reflection of the echo. Measuring to the mean height of a lunar area should improve the consistency of the range measurements and reduce the frequency of lunar range corrections imposed on the transmitter.

Time		T	$\Delta r$	General Area
Interval	L	В	(m)	on Moon
1/2 hour	-6.95 -7.11	-1.72 -1.72	40	Mosting
1/2 hour	-7.57 -7.63	-3.08 -3.04	-30	Mosting
1/2 hour	6.85 6.60	5.73 5.91	-10	Hyginus
1 day	1.43 1.44	6.78 6.69	-20	Ukert
1 month	-3.79 -3.78	1.90 1.71	0	Sinus Medii

Table 1
Range Variations of Several Locations on the Moon

Frequency measurements have been achieved by MIT (1) with accuracies of one part in  $10^{11}$  on the moon, while JPL (2) has obtained accuracies of the order of one part in  $10^{12}$  in measurements of frequency shift with transponders from the Lunar Orbiter series. The frequency spectrum of a lunar echo, as derived from an NRL cw lunar radar measurement, is shown in Fig. 4. If the spectrum from only the initial return is obtained, the deviations from a gaussian spectrum and the width will be reduced, thus improving the accuracy with which the centroid of the spectrum can be obtained.

## EFFECT OF SHIP'S VELOCITY ERROR ON POSITION LOCATION

Estimates of the ship's velocity may contain either random errors that vary with time or systematic errors that remain constant for several hours. Random errors with short periods (less than the observational interval) will spread the frequency spectrum and, for random velocities of less than 1 km/hr, will not materially affect the accuracy of the position determination. The effect of slowly drifting random errors of the ship's velocity can be reduced by obtaining several measurements in order to average the effective position error.

The systematic errors in the ship's velocity that can be tolerated without degrading the position accuracy are proportional to the effective error, as discussed in the error analysis section.

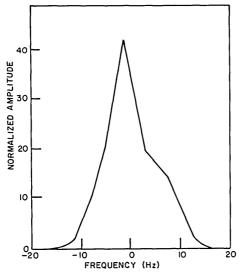


Fig. 4 - Frequency spectrum of a lunar radar echo

FREGENCY SPECTRUM OF A LUNAR RADAR ECHO

The effect of the ship's velocity error on the position accuracy has been derived in Appendix B as a function of the ratio of the velocity error to the measurement error. The results are given by

$$R_{1} = \frac{\sigma_{\Delta x_{v}}}{\sigma_{\Delta x_{m}}} = \frac{a_{12}(\sin^{2}\gamma + a_{12}^{2})^{1/2}}{(a_{22}^{2} + a_{12}^{2})^{1/2}} \frac{\sigma_{\Delta v/\omega_{e}}}{\sigma_{m}}$$
 (5)

and

$$R_2 = \frac{\sigma_{\Lambda y_w}}{\sigma_{\Delta y_m}} = \sin y \left(\sin^2 y + a_{12}^2\right)^{1/2} \frac{\sigma_{\Lambda v/\omega_e}}{\sigma_m}, \tag{6}$$

where

 $\gamma$  = the local hour angle,

 $a_{12} = \cos \gamma \sin \varphi - \cos \varphi \tan \delta$ ,

 $a_{22} = -\sin y \sin \varphi$ ,

 $\sigma_{\Delta x_v}$ ,  $\sigma_{\Delta y_v}$  = the errors caused by the velocity error on the longitude and latitude determination,

 $\sigma_{\Delta x_m}$ ,  $\sigma_{\Delta y_m}$  = the errors caused by the measurement errors on the longitude and latitude determination, and

 $\sigma_{\Delta\,v/\omega_e},~\sigma_{\rm m}$  = the velocity and measurement errors respectively.

Tables B1 through B7 represent  $R_1$  and  $R_2$  as a function of the local hour angle  $\gamma$ , the geocentric latitude  $\varphi$ , and the moon's declination  $\delta$  for the case when  $(\sigma_{\Lambda v'\omega_e})/\sigma_m=1$ . The total effective errors  $\sigma_{\Lambda x_T}$  and  $\sigma_{\Lambda y_T}$  of longitude and latitude are given by

$$\sigma_{\Delta x_T} = \sqrt{1 + R_1^2}$$

and

$$\sigma_{\Delta y_T} = \sqrt{1 + R_2^2} \ .$$

For velocity errors comparable to the measurement errors, the effective position error increases on the average by about 20 percent. For the assumed measurement error of  $\pm 300$  m, this corresponds to a velocity error of  $\pm 3$  cm/sec. If the velocity error exceeds the measurement error by a factor of four, then the total error is increased by 100 percent on the average. For larger velocity errors, observations over several hours are required to determine both the position and velocity corrections, if the full measurement accuracy is desired.

## DESIGN CONSIDERATIONS FOR LUNAR RADAR NAVIGATION SYSTEM

In considering the system parameters for a lunar bistatic radar system, it has been assumed that the operational measurement accuracy will be degraded by an order of magnitude from that obtained from lunar radar astronomy measurements (1-3). Furthermore for optimum efficiency, the time (range) and frequency measurement errors are made equal as discussed in the section on measurement errors. This corresponds to a 1- $\mu$ sec time resolution and a frequency resolution of one part in  $10^{10}$ , which are equivalent to a position error of  $\pm 300$  m.

Another consideration concerns the radar frequency to be employed with the lunar radar navigation system. Since one of the basic measurements is the doppler shift due to the relative motion of the ship's location with respect to the moon, the higher the radar frequency the greater the doppler shift and in turn the faster a readout can be obtained for a given accuracy. On the other hand, the complexity of a transmitting system that supplies sufficient energy to a dipole receiving system increases with increasing radar frequency. These considerations will require a compromise between the complexity of the radar transmitting system and the tolerable observation time. With larger receiving antennas, higher frequencies, or more complex transmitters, a higher accuracy and a more rapid readout can be obtained. An indication of the relation between these factors and frequency is given by

$$f_{\rm c} ~ \simeq \frac{1}{\epsilon_{\Delta f} ~ T_0 ~ A_r ~ G_t ~ P_t} ~, \label{eq:fc}$$

where

 $\epsilon_{\Lambda f}$  = the measurement accuracy in frequency,

 $T_0$  = the observation time,

 $A_r$  = the receiving aperture,

 $G_t$  = the gain of the transmitting antenna, and

 $P_{\rm r}$  = the transmitter power, and 500 MHz < f < 10,000 MHz for useful operation.

There are several basic requirements that the bistatic radar system has to satisfy:

1. A S/N ratio of 20 dB minimum is assumed at the receiver. If the receiver is to be made as simple as possible and to require no pointing by using a simple dipole as the receiving aperture, the major effort then has to be applied at the transmitter to provide sufficient energy at the receiving terminal. To obtain an indication of the type of transmitter that may be required, the parameters used in Table 2 are applied in the radar equation to supply the needed S/N ratio. If we assume a  $1000^{\circ}$ K receiver, the effective S/N is

$$S/N = \frac{P_t A_t \sigma G_r}{(4\pi)^2 R^4 KTB} \approx 100 ,$$

where

 $\sigma$  = the radar cross section of the moon,

 $G_r$  = the gain of a dipole,

R = the distance to the moon,

K = Boltzmann's constant,

 $\sigma = 4 \times 10^{-4} \times \pi b_0^2,$ 

 $b_0$  = the mean lunar radius, and

B = 5 Hz.

Table 2
Transmitter Characteristics for a Dipole Receiver

Characteristics	Quantity
Average Power Transmitted, $P_t$	2 MW
Antenna Diameter	170 m
Pulse-Compression Ratio	$2 \times 10^5$
Frequency Stability	1 part in $10^{12}$
Range Accuracy	1 μsec

If smaller transmitters are desired, some tradeoff between the receiver and transmitter can be made. For example, if instead of a dipole a 1-m antenna is used with the receiver and the wavelength is changed to 3 cm, the transmitting antenna need only be

30 m with an average power of 60 kW. This will also reduce the required readout time by a factor of 10.

- 2. The time at the transmitter and the receiver has to be synchronized within the basic measurement accuracy. Thus, for example, for an accuracy of 300 m, the two clocks have to be known within 1  $\mu$ sec. The use of atomic clocks is clearly indicated, and periodic resetting of the receiver clock is required. For a clock with a frequency stability of one part in  $10^{13}$  (thallium), the receiver clock would have to be reset every two months.
- 3. Special codes may be used to reduce the time of search in the uncertainty interval of the range measurements. In general, a search should be completed within a few minutes.
- 4. Special-purpose computers have to be available at both the transmitting station and the receiver to convert the known information of the lunar ephemeris and the position of the transmitter to range and range rate as a function of time. The expected receiver position is introduced to determine an uncertainty interval from which the corrected position can then be obtained.

Operationally, it is expected that the transmitter will continuously illuminate the moon, but that the coding interval and its start will be exactly controlled by the atomic clock of the transmitter. The receiver will then tune in on the received signal and search for its position. Once the position is found, the receiver will continue to track the position and indicate the motion of the ship or vehicle in latitude and longitude. The effective error of the indicated position can be simultaneously displayed by incorporating in the computer the information presented in the tables of Appendix A. Details of one possible receiver configuration are given in the next section.

## RECEIVING SYSTEM

The ultimate goal in designing and developing the receiving system is a package that provides automatic latitude and longitude information with little or no human adjustment. This concept is possible because all the tradeoffs for receiver simplicity will be vested in the transmitter. Since simplicity is of paramount importance, the design goal for the receiving antenna is a simple dipole or whip antenna. This eliminates the need for knowing where to point the antenna. The rest of the receiving system will utilize integrated circuits, microminaturization, and modular construction. It is envisioned that the final packaging of the receiver will be of the same volume and size as today's shipboard Loran C navigation receivers.

Figure 5 shows a proposed block diagram of the receiving system. Since biphase pulse compression is assumed in the transmitted signal, the receiver must decode the transmission in order to extract the necessary information. Also, the receiver must pass the signal through fairly narrow filters, which requires that the local oscillator (LO) chain have coarse frequency tracking to correct for the earth's rotation. This is achieved by squaring the i-f signal to remove the biphase code and then, with a frequency-following filter and appropriate counters, developing the control signals for adjusting the LO so that the signal will pass through the narrow filters used.

Once the LO frequency is correctly set, the signal is fed to ten correlators and filters (the number required is determined by operational requirements) to determine the

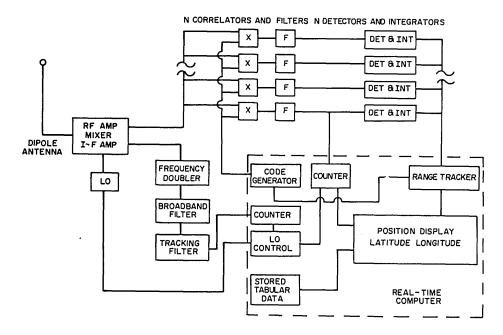


Fig. 5 - Navigation receiver

range or time of flight of the signal. The code generator will develop the proper codes which will be correlated with the incoming signal to provide coarse range information. The range tracker must sense a signal; otherwise, it will command the code generator to increment the code until a signal is found. Once the signal is found, the range tracker switches into the fine range mode to measure the range or time specified. Also, fine frequency will be determined after the fine range is established, and this is accomplished by using the output of one or more range channels and feeding its signal to an axiscrossing counter to determine precisely the frequency of the received signal. The range and frequency will be fed into a computer, which will display latitude and longitude automatically. The acquisition time is determined by the required range and frequency resolutions and the initial uncertainty of the position under observation. Assuming  $1-\mu \sec$  range precision and 100-km position uncertainty, the acquisition time is

	Maximum (sec)	Average (sec)
Lock in coarse frequency	40	20
Lock in coarse range	60	30
Lock in fine range	100	50
Measure frequency	100	100
Total time for acquisition	300	200

Once the signal has been acquired, updating in range is fairly quick (10 sec). However, to determine the frequency will still require 100 sec. The times used are for a particular

set of parameters. Tradeoffs in performance and system complexity will alter the acquisition and updating times required.

#### DISCUSSION AND CONCLUSIONS

To evaluate the performance of the lunar radar navigation system, it may help to compare it with the Transit system, which is similar in fundamental concept. The differences of the lunar navigation system with respect to Transit can be summarized as follows:

- 1. A natural satellite is employed as the reference point in the sky. This obviates the necessity of periodically injecting artificial satellites for this purpose, but restricts to some extent the coverage, as a function of time and position, which would be available with a sufficient number of artificial satellites.
- 2. The orbit of the moon is well known, and any minor corrections to its orbit can be made by the transmitter, which continuously monitors the motion of the moon. This makes it possible to increase the effective accuracy of the range and range rate data as compared with Transit, where the orbit of the satellite has to be updated and then predicted for a period of several hours.
- 3. The relatively slow rotation of the earth as compared to the rapid motion of the satellite requires a higher frequency stability at the receiver for a comparable accuracy in position measurements. This fact also makes the position determination more sensitive to errors in the ship's velocity.
  - 4. One-way communication is available with some minor modification.
- 5. The moon is not vulnerable to hostile interference and is much more difficult to jam.
- 6. Finally, due to the large distance of the moon from the earth, the system may be considered in the future as a navigational aid to lunar and close interplanetary spacecrafts.

At present, the achievable accuracy is limited technologically by the accuracy of the available atomic clocks and the need to reset them periodically. With the future development of atomic clocks and the placement of transponders on the moon, accuracies of several meters should be possible. On the other hand, for stationary locations where observations can be made for several hours, atomic clocks can be reset automatically and simultaneously with the position measurements.

To evaluate more closely this concept, it will be necessary to utilize one of the many radar transmitters now available for testing the search and track mode of a receiver in an assumed unknown location and to establish more definite performance criteria. If this concept should prove itself practical, it may then be feasible to consider placing a transponder or a corner reflector on the moon to increase the accuracy as well as reduce the time of observation.

Future improvements of the lunar radar navigation concept may include the utilization of additional radio reference points in the sky. The additional radio reference points can be made available by placing isotropic reflectors at the libration points of the moon.

The applicable libration points are located in the moon's orbital plane and consist of the two stable equilibrium points  $L_4$  and  $L_5$  (4) (equilateral points) at  $\pm 60^{\circ}$  in right ascension relative to the moon, and one semistable equilibrium point  $L_3$  (collinear point) opposite the moon. If a reflector is injected at any one of these points, it is expected to stay indefinitely in the general vicinity of the particular equilibrium point. The small periodic motions of the reflectors around the equilibrium points can be compensated for by the transmitter corrections as before. Utilization of these reference points could provide (a) independent determination of position and velocity and reduction of the geometrical dilution of the measurement errors, (b) measurement of the three orthogonal components in both position and velocity for limited coverage, and (c) an increase of coverage to 24 hours for two coordinates. The ability to determine three coordinates in both position and velocity may be of particular use when the concept is applied as an independent navigation system for surface, air, or spaceborne vehicles, but the extent of coverage still has to be investigated.

#### ACKNOWLEDGMENTS

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## Appendix A

## ERROR ANALYSIS OF POSITION DETERMINATIONS

The effective error as a function of time and position can be derived from the basic relation of the geometric configuration of the earth-moon system (Fig. A1). Let the distance from the unknown location to the moon be expressed by

$$\rho = -b + \sqrt{s^2 + r^2 - 2 \, \text{sr cos } \beta} \,\, . \tag{A1}$$

where

- b is the lunar radius at the point of reflection,
- s the earth-moon center-to-center distance,
- r the geocentric radius of the location, and

$$\cos \beta = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos \gamma$$
.

In the latter equation,

- φ is the geocentric latitude of the unknown location,
- $\delta$  the moon's declination, and
- $\gamma$  the local hour angle =  $\theta \alpha \lambda$ ,

where

- $\theta$  is the Greenwich sidereal time,
- $\alpha$  the moon's right ascension, and
- $\lambda$  the longitude of the unknown location.

If we assume the moon to be stationary, then the relative velocity of the point under observation is

$$\dot{\rho} = -\frac{sr}{\rho + b} \frac{d(\cos \beta)}{dt} = \frac{sr}{\rho + b} \omega_e \cos \varphi \cos \delta \sin \gamma , \qquad (A2)$$

where  $\omega_e = d\gamma/dt$  is the earth's rotation in radians per second.

Any error in longitude  $\lambda$  or latitude  $\phi$  will produce an error in  $\rho$  and  $\dot{\rho}$ , which to first-order accuracy can be expressed as

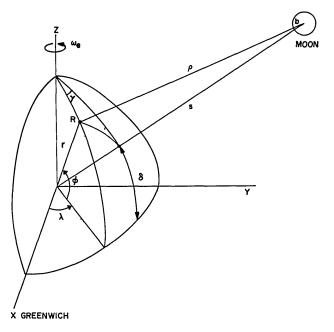


Fig. A1 - Geometry of the earth-moon system

$$\Delta \rho = -r \cos \varphi \cos \delta \sin \gamma \Delta \lambda$$

$$-r(\cos \varphi \sin \delta - \sin \varphi \cos \delta \cos \gamma) \Delta \varphi \tag{A3}$$

and

$$\Delta \dot{\rho} = -r \omega_e \cos \varphi \cos \delta \cos \gamma \Delta \lambda$$

$$-r \omega_e \sin \varphi \cos \delta \sin \gamma \Delta \varphi. \tag{A4}$$

Converting the angular errors to linear errors by setting  $\Delta x \approx r \cos \varphi \Delta \lambda$  and  $\Delta y = r \Delta \varphi$ , the errors in position  $\Delta x$  and  $\Delta y$  are related to the differentials  $\Delta \rho$  and  $\Delta \dot{\rho}$  by Eqs. (A5) and (A6):

$$\frac{\Delta \rho}{\cos \delta} = -\sin \gamma \, \Delta x + (\cos \gamma \, \sin \varphi - \cos \varphi \, \tan \delta) \, \Delta y \tag{A5}$$

$$\frac{\Delta \dot{\rho}}{\omega_{e} \cos \delta} = -\cos y \, \Delta x - \sin y \, \sin \varphi \, \Delta y \, . \tag{A6}$$

It is seen that the effective measurement error is a weak function of the moon's declination and for extreme values of  $\delta = \pm 28^{\circ}$  will increase the basic measurement error by about 10 percent. To obtain the corrections of  $\Delta x$  and  $\Delta y$ , we have to solve Eqs. (A5) and (A6). Let

$$\begin{split} m_1 &= \frac{\Delta \rho}{\cos \delta} \;, \qquad m_2 &= \frac{\Delta \dot{\rho}}{\omega_e \cos \delta} \;, \\ a_{11} &= -\sin \gamma \;, \quad a_{12} &= \sin \phi \cos \gamma - \cos \phi \tan \delta \\ a_{21} &= -\cos \gamma \;, \quad a_{22} &= -\sin \phi \sin \gamma \;. \end{split}$$

Then

$$\Delta x = \frac{\begin{vmatrix} m_1 & a_{12} \\ m_2 & a_{22} \end{vmatrix}}{\Delta} = \frac{m_1 a_{22} - m_2 a_{12}}{\Delta} , \qquad (A7)$$

$$\Delta y = \frac{m_2 a_{11} - m_1 a_{21}}{\Lambda} \,, \tag{A8}$$

where  $\Delta = a_{11}a_{22} - a_{21}a_{12}$ .

The variances of the corrections  $\Delta x$  and  $\Delta y$  in terms of the measurement variances  $\sigma_{m_1}^2$  and  $\sigma_{m_2}^2$  are given by

$$\sigma_{\Delta x}^{2} = \left(\frac{\partial \Delta x}{\partial m_{1}}\right)^{2} \quad \sigma_{m_{1}}^{2} + \left(\frac{\partial \Delta x}{\partial m_{2}}\right)^{2} \quad \sigma_{m_{2}}^{2}$$

$$= \frac{1}{\Delta^{2}} \left(a_{22}^{2} \quad \sigma_{m_{1}}^{2} + a_{12}^{2} \quad \sigma_{m_{2}}^{2}\right) \tag{A9}$$

$$\sigma_{\Delta y}^2 = \frac{1}{\Lambda^2} \left( a_{21}^2 \ \sigma_{m_1}^2 + a_{11}^2 \ \sigma_{m_2}^2 \right) \tag{A10}$$

For efficient operation, the measurement variances are equal:  $\sigma_m^2 = \sigma_m^2 = \sigma_m^2$ . Then the standard deviations of the corrections of  $\Delta x$  and  $\Delta y$  are

$$\sigma_{\Delta x} = \frac{1}{\Lambda} \sqrt{a_{22}^2 + a_{12}^2} \ \sigma_m \tag{A11}$$

and

$$\sigma_{\Lambda V} = \frac{1}{\Lambda} \sqrt{a_{21}^2 + a_{11}^2} \ \sigma_m \ , \tag{A12}$$

where

$$\Delta = + \sin^2 \gamma \sin \varphi + \cos \gamma^2 \sin \varphi - \cos \gamma \cos \varphi \tan \delta$$
$$= \sin \varphi - \cos \gamma \cos \varphi \tan \delta ,$$

$$\sqrt{a_{22}^2 + a_{12}^2} = (\sin^2 \phi \sin^2 \gamma + \sin^2 \phi \cos^2 \gamma + \cos^2 \phi \tan^2 \delta - 2 \sin^2 \phi \cos \phi \tan \delta \cos \gamma)^{1/2},$$

and

$$\sqrt{a_{11}^2 + a_{21}^2} = \sqrt{\sin^2 \gamma + \cos^2 \gamma} = 1$$
.

Therefore,

$$\sigma_{\Delta x} = \frac{\sqrt{\sin^2 \varphi + \cos^2 \varphi \tan^2 \delta - \sin 2 \varphi \tan \delta \cos \gamma}}{\sin \varphi - \cos \gamma \cos \varphi \tan \delta} \sigma_m , \qquad (A13)$$

$$\sigma_{\Delta y} = \frac{1}{\sin \varphi - \cos y \cos \varphi \tan \delta} \sigma_m . \tag{A14}$$

These expressions will give the effective errors of  $\Delta x$  and  $\Delta y$  as a function of the latitude  $\varphi$ , the local hour angle  $\gamma$ , and the moon's declination  $\delta$ . The computer printout in this appendix gives the program for solving Eqs. (A13) and (A14).

The equations are computed for various values of  $\delta$ ,  $\varphi$ , and  $\gamma$  in Tables A1 through A7 for  $\sigma_m$  = 1/cos  $\delta$ . The tables marked "a" indicate the effective error in longitude, and those marked "b" give the corresponding error in latitude. The first vertical column gives the latitude of the observed position from +90° to -90° in 10° intervals. The top row gives the hour angle in 10° intervals (40 minutes of time) starting at transit. The tables extend symmetrically for negative hour angles (not shown). Each number, when multiplied by the given basic measurement error, indicates the effective error in meters. The tables are separated by about two days in time, corresponding to a change in declination by 9°. The effective errors increase as the latitude of the observed position approaches the declination of the moon.

## Program for Solution of Eqs. (A13) and (A14)

TN5,4A 09/03/68

```
PROGRAM COVERAGE
    DIMENSION SIGX(19,7,19), SIGY(19,7,19), IPRINT(3)
    CCMMGN/1/SIGX
    COMMON/2/SIGY
    TYPE INTEGER DCLN, HA
    TYPE DOUBLE
                  DEC, PHI, GAMMA, TAND, SINP, COSP, COSG, DELTA, COSD, SIND
    TYPE DOUBLE NUMERATE
    DATA (IPRINT = 8H(*+*,5X,,0,6HF4,1))
900 FORMAT (*1
                       ERROR IN LONGITUDE
                                               FCR
                                                        DEC =*, [4,/)
901 FORMAT (+0+,5X,+HA(+OR+) 0 10 20
                                             30 40
                                                       50 60
                                                                  70
                                                                        80
* 90 100 110 120 130 140 150 160 170 902 FORMAT (*+*, 15)
                                                     180+,/,*
                                                                 PHI +)
903 FORMAT (*1
                       ERROR IN LATITUDE
                                               FOR
                                                        DEC =*, 14,/)
904 FORMAT (+0+)
110 FORMAT (*5X,*,12,*(1X*)
    DTR = 0.017453292519943296D
    DO 100
             J=1.7
    DCLN = 9*(1-J) + 27
    DEC = DCLN+DTR
    COSD = DCOS(DEC)
    SIND = DSIN(DEC)
    FAND = SIND/COSD
    D0 50 I = 1,19
LAT = 10*(1-I) + 90
    PHI = LAT+DTR
    SINP = DSIN(PHI)
    COSP = DCOS(PHI)
    D0 30
            K = 1,19
    HA = 10*(K - 1)
    GAMMA = HA*DTR
    COSG = DCOS(GAMMA)
    IF(SINP*SIND + COSP*COSD*COSG ,GT, 0) GO TO 10
    SIGX(I,J,K) = SIGY(I,J,K) = 0.
    GO TO 30
 10 IF(LAT .NE. DCLN .0 COSG = DCOS(0.1*DTR)
                        OR.
                            HA .NE. 0)
                                           GØ TØ 20
 20 DELTA = COSP * TAND * CCSG = SINP
    NUMERATE = DSQRT(SINP**2 + (COSP*TAND)**2 - 2*SINP*COSP*TAND*COSG)
                       DELTA = 1.00-30
    IF(DELTA .EQ. 0)
    IF (NUMERATE .EQ. U.)
                            NUMERATR = 1.00-30
    SIGX(I,J,K) = NUMERATR/DELTA
    SIGY(I.J.K) = DABS(1./DELTA)/COSD
    SIGX(I,J,K) = ABSF(SIGX(I,J,K))/COSD
 30 CONTINUE
 50 CONTINUE
    PRINT 900,
               DCLN
    PRINT 901
    00 35
            I = 1.19
    LAT = 10*(1 - I) + 90
    PRINT 902, LAT
    M = 0
    00 33
           K=1,19
    !F(SIGX(I,J,K) ,EQ, 0,)
                              GO TO 34
 33 M = M + 1
 34 IF(M ,EQ, 0) G0 T0 35
    ENCODE (8,510, IPRINT(2)) M
```

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```
PRINT IPRINT,
                     (SIGX([,J,K),K=1,M)
 35 PRINT 904
    PRINT 903, DCLN
    PRINT 901
    DO 40 I = 1,19
LAT = 10+(1-I)+90
    PRINT 902, LAT
    M = 0
    DO 38 K=1,19
    IF(SIGY(I,J,K) .EG, 0,) G0 T0 39
 38 M = M + 1
 39 IF(M ,EQ, 0) G0 T0 40 ENCODE (8,110,IPRINT(2)) M
    PRINT IPRINT,
                    (SigY(I,J,K),K=1,M)
40. PRINT, 904
100 CONTINUE
    END
```

Table A1a Error in Longitude for a Declination Angle of  $27^\circ$ 

A F F	ERROR 1	NO J	IN LONGITUDE	ĮIJ.	F 0R	DEC	# 	27				)							
HA (+0R=	3= 0	10	20	30	4 0	50	69	70	80	06	100	110	120	130	140	150	160	170	180
T 0	11	년 대	1,1	न <del>।</del>			1.1	4	1,1	ਜ਼ ਜ਼	1,1	4.	1,1	4,4	1,1	£, £	1,1	1,1	1.1
60	1,1	#1 #1	1,1	T t		1.1	1,1	44	1.1	۲. ۲	1,1	+ +	1,1	<b>1</b> • 1	1,1	1,1	1,1	1,1	1,1
7.0	1,1	1.1	1,1	स्र	1,1	ŗ,	1.1	1,1	1,1	4,4	1,1	1,1	1,1	1,1	1.1	4,4	1,1	1,1	1,1
60	+ <del>+</del> +	ਜ ਜ	1,1	1,1			1.2		1,2	1,2	1,2	1,2	1,1	ਜ <b>਼</b>	1,1	ਜ਼ : ਜ			
50	, <del>, ,</del>	1,1	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2						
40	4.4	1,2	1,2	4.3		4.4	1.4	4 4	주· 단	1,3	1,3	4.2							
30	ून न	1,7	2,3	2	2,3	2,1	1.9	1,7	1,6	1,5	4.4								
20	न न	1.3	2.0	3,9	14.0	12,1	4.7	3,0	2,3	. <del>1</del>	1.7								
10	1,1	1,2	1,3	1,6	2,1	3,1 1,0		*	6,5	3,4									
0	त	ਜ਼ <b>਼</b> ਜ	1,2	1,3	1,5	1,7		8,3	6.5	*									
-10	T • T	1,1	1,2	1,2		4		1,9	2.4										
920	1,1	1.1	1,1	1,2	1,2	1, 1	4,1	1,5											
0 S =	1,1	1.1	T. T.	1,2	1,2	1,2	1,3	5,1											
04.	T • T	<del>्</del> स	4.4	1,1	1,2	1.2	1,2	,											
05°	1,1	਼ਜ਼	1,1	1,1		1,2													
096	1,1	स स	1.1					4 :											
e70																			
0 8 6									•										
0.5e																			

Table A1b Error in Latitude for a Declination Angle of  $27^\circ$ 

HA(+0R=) Ph.I 50 1	+0R=) U 10 20	1 10	20	50	40	50	60	70	80	90	100	110	120	130	140	150	160	0 +	
60	1,3	1,3	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,1	1,1	1,1	1,1	<b>ਜ</b> •ਜ	1,1	4	-	1 1,1	
7.0	1,5	1,5	1.4	1,4	1,4	1,4	1,4	1,3	1.2	1,2	1,2	च च	1,1	4	1.0	100	•	1,0	
6.0	88 	1,3	1.8	1,7	1,7	1,6	4,5	1.4	4.4	1,3	1,2	1,2	1,1	ਜ. ਜ	1,1	1,0	_		
īv O	5,6	2,5	2,4	2,3	4,2	2,0	1.9	1,7	1,6	4,5	1.4	1,3	1,2						
4 0	4	4 4,3	3 4,1	3,7	5,3	5,9	2.2	2,2	2.0	1,7	1,6	<b>†</b>							
0 8	19,1	1 17,1	13,1	6 2	6.9	21.5	4.0	3,2	2,7	2,2	1,9								
20	8,2	2 8,7	7 10,4	15,5	45.0	32,8	10.9	6,3	£ . 4	3,3	5,6								
10	4,8	3,5	3,8	£ , 4	5,3	7.5	14,5	*	13,0	6,5									
0	2,2	2 2,2	2 2,3	2,5	2,9	4.6	4.	4.4	12,7	*									
010	1,7	7 1.7	7 1,7	2,1	2,0	2,3	2.6	3,3	4,3										
=20	1,4	4 1,4	4 1,4	1.5	1,6	1,7	4.5	2,2											
900	1,2	2 1,2	2 1,2	1,3	1,3	1.4	1.6	1,7											
e 40	1,1	1 1,1	1 1,1	111	1,2	1,3	1,3												
0.00	1,0	0 1.0	0 1,0	1,1	1,1	1,1													
09=	1,0	0 1.0	0 1.0	_															
ç																			

	Angle of 18°
ole A2a	· a Declination
Table	$\mathbf{for}$
	Longitude
	r in ]
	or

				田	rror i	n Lon	Table A2a Error in Longitude for a Declination Angle of $18^\circ$	Table A2a for a <sub>t</sub> Decl	A2a Declin	ation,	Angle	of 18°							
æ	ERROR IN		LONGITUDE		R.O.R	DEC	11 12	18											
HA (+0R=)	R=) 0	, 0 1	20	30	40	50	09	70	80	06	100	110	120	130	140	150	160	170	180
1 ± C 2 0 0	1,1	1,1	1.	1,1	1,1	ਜ	1,1	1,1	1.1	1,1	1,1	<b>+</b> +	1,1	다. 다	1,1	4.4	1,1	41	년 년
8.0	1,1	1,1	1.1	1,1	1.1	1,1	1,1	1,1	1.1	1,1	1,1	4,4	1,1	۲. د د	1.1	1.1	1,1	1,1	1.1
20	1,1	1,1	1.1	ਜ • ਜ	1.1	101	1.1	1,1	1,1	11	1.1	4.4	1,1	1,1	1.1	# + #			
09	ਂ ਜ <b>਼</b>	1,1	1,1	1,1	1,1	4.4	1,1	1,1	<b>T</b> • T	1,1	1,1	# # #	1,1						
50	1,1	न	1,1	, F • T	1,1	1,1	1,1	1,1	1,1	<b>#</b>	1,1	F • F							
04	1,1	1,1	1.1	ਜ <b>਼</b>	1,1	1,1	1+1	न्। न	1.1	1,1	1.1								
30	1,1	न <b>*</b> न	1,1	1,2	1,2	H.	7,4	1.3	1,2	1.2	1.2								
20	т. т	1,7	2,3	2,3	2,2	2,0	1.8	1.6	1.5	4,1									
01	1,1	т. т	1.4	1.9	3,2	8,1	21.4	5,0	3,0	2,2									
0	1,1	4,4	1,1	1,2	1.4	1.6	2+1	3,1	6,1	*									
610	1,1	T.	1,1	नं न न	1,2	1,3	1,4	1,5	1,8										
420	1. 1.	1,1	1,1	स	1,1	1,1	1,2	1,2	1,3										
020	11	त. च	1,1	1.1	1,1	4.4	T • T	1,2											
040	T .	1,1	4,4	t = T	1,1	ਜ <b>਼</b>	ਜ• ਜ	ਜ ਜ											
e 50	1,1	4	1,1	1,1	त न	1,1	T • T												
098	1,1	1,1	1.1	1.1	1,1	T + T													
01e	1,1	1.1	1,1																
980																			
06-																			

					Table A2b Error in Latitude for a Declination Angle of $18^{\circ}$	in Lat	itude	Table A2b for a Decli	A2b Declin	ation 1	Angle	of 18°						
	ERROR IN LATITUDE	N LAT	TUBE		F 0R	DEC	я U	8										
H	HA(+0Rs) 0	10	20	30	4 0	20	9	70	80	9.0	100	110	120	130	140	150	100	170
7 50 100	1,1	1,1	1,1	1,1	1.1	1,1	1,1	1,1	1.1	ਜ ਜ	1,1	## ##	1,1	1,1	1,1	4	1,1	11
80	1,1	1,1	1.1	1.1	1,1	1,1	1.1	1,1	1,1	1,1	1,1	1.0	1,0	1.0	1.0	1.0	1.0	1,0
7.0	1,3	1,3	H . H	1,2	1,2	1.2	1,2	1,2	1.1	4,4	1,1	<b>†•</b>	1,1	1,0	1.0	4.0		
60	1,5	1,5	1,5	1.4	1.4	4.4	1,3	1,3	1,3	4.2	1,2	♥ e	1.1					
50	1,9	1,9	1.8	1.8	1,7	1,7	1,6	4.5	4.4	4.	1,3	1.3						
0	2,7	2,6	2.6	2,5	2,3	2,2	2,0	1,9	1.8	1,6	1,5							
30	4	4.7	4.5	4.1	3,7	3,3	5,9	5,6	2,3	2,1	1.9							
20	28,7	25,4	19.1	13,5	6.7	7,2	2,6	4.	3.6	3,4								
10	7,2	7,4	8	10,2	14.7	32,8	77,0	16,4	8.5	6,1								
0	3,2	3,3	4.6	3.7	4.2	5,0	6,5	9,5	18,6	*								
-10	2,1	2,2	2,2	2,3	2,5	2,8	3	3,7	4.6									
-20	1,6	1,6	1.7	1,7	1,8	2,0	2,1	2.4	2.7									
930	1,3	4.1	1.4	1,4	1,5	1,5	1,6	1,8										
0 4 0	1,2	1,2	1,2	1.2	1,3	1,3	1,4	1,4										
150	1,1	1,1	1,1	<b>+</b> + +	1,1	1,2	1,2											
09=	1,0	1,0	1,0	1,0	1,1	न्। भ	•											
.70	1,0	1.0	1.0															
0 8 9																		
0																		

Table A3a Error in Longitude for a Declination Angle of  $9^\circ$ 

					Er	ror in	Error in Longitude for a	itude f	or a I	Seclina	Declination Angle	ngle (	$_{ m o6}$ Jo				
	ERROR IN	_1	ONG! TUDE		FOR	DEC	II	ο.									
2	HA(+0R=) 0	10	20	30	40	20	9	. 70	90	06	100	110	120	130	140	150	160
t Ø	1,0	, <del>1</del>	1.0	1,0	1,0	1,0	1,0	1,0	1.0	1,0	1.0	1,0	1,0	1,0	1.0	1.0	1,0
80	1.0	0 1	1,0	1.0	1.0	1,0	1,0	1,0	1.0	4.0	1.0	1.0	1,0	1.0	1.0	1.0	
70	0 <b>•</b> ਜ਼	. <del>1</del>	1.0	1,0	1,0	10	1,0	1,0	4.0	1,0	1.0	1,0					
60	1.0	1,0	1.0	1.0	1,0	1.0	1.0	1,0	1.0	1,0	1.0						
20	1.0	1,0	1.0	1,0	1.0	1,0	1.0	1,0	1.0	1,0	1,0						
0	1,0	1,0	1,0	1.0	1,0	1,0	1,0	1,0	1,0	1,0							
30	1,0	1,0	1,0	1.0	1.0	4,0	1,1	1,1	1,1	1,0							
20	1.0	1,0	1.0	1,1	ू. इ.	1,1	1,1	1,1	1,1	ਜ <b>ਂ</b> ਜ							
10	1.0	1,7	2,2	2,3	2,1	1,9	1.8	1,6	1,5	4.1							
0	1.0	1,0	1,1	1,2	1,3	1,6	2,0	3,0	5,8	* * *							
*10	1.0	1,0	.0 ∓	1,0	1,1	1,1	1+1	1,2	1,3								
•20	1,0	1.0	1.0	1.0	1.0	1,0	ਜ <b>਼</b>	f.	1,1								
930	1,0	1,0	1.0	1,0	1.0	1.0	1.0	1,0	1.0								
040	1,0	1,0	1.0	1.0	1.0	1,0	1,0	1,0	1.0								
o 75 0	1,0	1.0	1,0	1,0	1.0	1,0	1,0	1,0									
09=	1,0	1,0	1.0	1.0	1,0	1,0	1.0	1,0									
e70	0 ° É	1,0	1.0	٠ •	1,0	1,0	1,0										
980	1,0	1,0	1.0														
0.5 =																	

Table A3b Error in Latitude for a Declination Angle of  $9^\circ$ 

	ERROR 1	IN LAT	LATITUDE		FOR	DEC	။ ပ	Φ.				,							
•	HA(+0R=) 0	01	20	30	4	20	9	70	80	06	100	110	120	130	140	150	160	170	180
1 6	1,0	1,0	1.0	1,0	1.0	4,0	1,0	1,0	1,0	1,0	1,0	1.0	1,0	0.1	1.0	1,0	1,0	1,0	1,0
80	1,1	1+1	1,1	1,1	1,1	1,0	1,0	1,0	1.0	1,0	1.0	1.0	1.0	1.0	1.0	1,0			
7.0	1,1	1,1	1.1	1.1	1.1	1.1	1,1	1,1	1,1	1,1	1,1	T . T							
60	1,3	1,3	1,3	1,3	1,3	1,2	1.2	1,2	1,2	1,2	1,2								
ربر 0	1,5	1,5	1,5	1,5	1,5	4,1	1.4	1,4	1,4	1,3	1,3								
40	1,9	1,9	1,9	1,9	1,8	1,8	1.7	1,7	1,6	1,6									
30	2,8	2,8	2,7	2.7	5,6	2,5	2,3	2,2	2,1	2,0									
20	5,2	5,2	5,0	4. 30	4.	4,1	3,8	υ 5	3.2	3,0									
10	57,3	50,5	37,4	26,3	18,7	13,8	10.6	4.	6,9	5,8									
0	4, 6	6,5	6,8	7,4	8,3	6.6	12,8	18,7	36,8	*									
a10	3,1	3,1	3,2	3,3	3,5	3,7	4.0	4,5	5.0										
<b>620</b>	2,1	2,1	2,1	2,2	2,2	2,3	2,4	2,6	2.8										
0 K) B	1,6	1,6	1,6	1,6	1.7	1,7	1. 8	1,9	1,9										
040	1,3	1,3	1,3	4.4	4.1	1,4	1.4	1,5	1,5										
150	1,2	1,2	1,2	1,2	1,2	1,2	1,2	<b>1</b>								,			
099	1,1	<b>#</b>	1,1	नं न	1,1	T i	1,1	1,1											
07c	1,0	1,0	1,0	1,0	1.0	1,0	1.0												
089	1,0	1,0	1.0																
06-																			

Table A4a

Table A4a Error in Longitude for a Declination Angle of 0° NGITUDE FCR DEC = 0	0 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 3	0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0	0 1,0 1,0 1,0 1,0 1,0 1,0 1,0	0 1.0 1,0 1,0 1,0 1,0 1,0 1,0 1,0	0 1,0 1,0 1,0 1,0 1,0 1,0 1,0 1.0	0 1,0 1,0 1,0 1,0 1,0 1,0 1,0 1,0	0 1,0 1,0 1,0 1,0 1,0 1,0 1,0	0 1,0 1,0 1,0 1,0 1,0 1,0 1,0	0 1,0 1,0 1,0 1,0 1,0 1,0 1,0	0 1,0 1,0 1,0 1,0 1,0 1,0 1,0	0 1,0 1,0 1,0 1,0 1,0 1,0 1,0 1,0	0 1,0 1,0 1,0 1,0 1,0 1,0 1,0	0 1,0 1,0 1,0 1,0 1,0 1,0 1,0	0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0	0 1.0 1.0 1.0 1.0 1.0 1.0 1.0	0 1,0 1.0 1,0 1,0 1,0 1,0 1,0	0 1,0 1,0 1,0 1,0 1,0 1,0 1,0 1,0	0 1,0 1,0 1,0 1,0 1,0 1,0 1,0	•
F.	4																		1,0 1,0
IN LONGITUD	10	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0 + 4	1.0	1.0	1,0	1.0 1.0
ERROR I		F1 90 1,0	80 1,0	70 1,0	60 1,0	50 1,0	40 1,0	30 1,0	20 1,0	10 1,0	0 1,0	10 1,0	20 1.0	30 1,0	40 1,0	50 1,0	60 1,0	70 1,0	80 1.0

Table A4b Error in Latitude for a Declination Angle of  $0^{\circ}$ 

	180																			
	170 1																			
	160 1																			
	150																			
	140																			
	130																			
	120																			
	110																			
	100																			
	06	1,0	1,0	1,1	1,2	1,3	4,6	2,0	6,5	5,8	*	5,8	5,9	2,0	1,6	1,3	1,2	ਜ <b>ਜ</b>	1,0	1,0
	80	1.0	1.0	1,1	1.2	1,3	1.6	2.0	2.9	5.8	* * * * * * * * * * * * * * * * * * * *	5.8	5.9	2.0	1.6	1,3	1,2	1,1	1.0	1,0
0	7.0	1.0	1,0	् न द	1,2	1,3	1,6	2,0	2,9	5,8	* * * *	5,8	5,9	2.0	1,6	1,3	1,2	1,1	1,0	1.0
н	09	1,0	1.0	1,1	1,2	1,3	1,6	2,0 2	2.9	8.6	* * * * *	5.	2,9	2,3	1,6	1,3	1,2	: + t	υ, τ	1,0
DEC	20																			
	ď	1.0	4.0	1,1	1,2	1,3	4,6	2,0	2,9	5,8	*	5,8	2,9	2,0	1,6	4.4	1,2	1,1	1,0	1,0
F OR	4 0	1.0	1.0	1,1	1,2	1,3	1,6	2,0	2,9	5.8	*	5,8	2,9	2,0	1,6	1,3	1,2	1,1	1,0	1.0
	30	1,0	1.0	1,1	1,2	1.3	1,6	2,6	5,9	5,8	*	5,8	519	2.0	1,6	1,3	1,2	4	1,0	1.0
UDE	20	1.0	1.0	1,1	1.2	1,3	1,6	2,0	5.9	5,8	*	5,8	2,9	2,0	1,6	1,3	1,2	1,1	1,0	1.0
LATIT	10	1,0	1,0	1,1	1,2	1,13	1,6	2.0	5,9	بر. ج	* * * *	5,8	6 2	2,0	1,6	ب ب	1,2	1,1	1,0	1,0
Z	0	1,0	1,0	1,1	1,2	1,3	1,6	2,0	2,9	5. 8	* * *	5,8	2,9	2,0	1,6	1,3	1,2	1,1	1,0	1,0
ERROR IN LATITUDE	HA (+0R=)	4	ਜ	<b>ન</b>	ਜ	ਜ	+1	C)	N	īV	*	'n	(VI	(V	4	ᆏ	-	**	-	**
		1 0 0 5 0	8.0	7.0	60	ເນ O	4 0	30	20	10	0	<b>a</b> 10	#20	029	0.40	0 10 10	±60	a70	09€	063

Table A5a Error in Longitude for a Declination Angle of  $\text{--}9^\circ$ 

N N	ONG	LONGITUDE		r CR	DEC	н	6											
10 20 3	 O		9	40	20	0.9	70	80	06	100	110	120	130	140	150	160	170	180
1,0 1,0	•					•												
1,0 1,0 1,0	-1	) <del>-</del>	_	1.0	10	1,0												
1,0 1,0 1.0	4	1.0		1.0	1,0	1.0	1,0											
1,0 1,0 1.0	ਜ	1.0		1.0	1.0	1.0	1,0											
1,0 1,0 1,0	т о	1.0		1,0	1.0	1.0	1,0	1.0										
1,0 1,0 1,0		1.0		1,0	1,0	1,0	1,0	1,0										
1,0 1,0 1,0	+	1.0		1.0	0.4	1.1	1,1	1.1										
1,0 1,0 1,0	41	1.0		1.1	1,1	1,1	1,2	1,3										
1,0 1,1 1,2	ਜ ਜ	1,2		1,3	1.6	2,0	3,0	ۍ ه	* * *									
1,7 2,2 2,3	C4	2,3		2,1	6.1	9,1	1,6	1,5	1,4									
1,0 1.0 1,1	4	+1		1,1	۲.	1,1	1,1	1	1,1									
1,0 1,0 1,0	1	1.0		1,0	1.0	1,1	1,1	1.1	1,0									
1,0 1,0 1,0	ਜ	1,0		1,0	1.0	1,0	1,0	1.0	1,0									
1,0 1,0 1,0	٠.	1,0		1.0	1,0	1.0	1,0	1,0	1,0	1.0								
1,0 1,0 1,0	4	0.		1,0	1,0	1.0	1,0	1.0	1,0	1.0								
1.0 1,0 1,0	о. Н	1,0		1.0	1,0	1,0	1,0	1,0	1.0	1.0	1.0							
1,0 1,0 1,0		1.0		1.0	1.0	1.0	1.0	1,0	1,0	1.0	0.4	1,0	1.0	1.0	1.0			
0,1 0,1 0,.	1 0.	1,0		1,0	1.0	1,0	1,0	1,0	1,0	1.0	D.	1,0	1.0	1.0	1.0	1,0	1,0	1.0

Table A5b

	170 180																			1,0 1,0
	160 1																			1,0
	150																		1,0	1,0
	140																		1.0	1.0
	130																		1.0	1,0
ה ו	120																		1.0	1,0
10 alz	110																	1,1	1.0	4.0
on Aug	100															1,3	1,2	1,1	1.0	1.0
linad	06										* *	5,8	3,0	2,0	1,6	1,3	1,2	1,1	1,0	1,0
a Dec	90						1,5	1,9	2,8	5,0	36,8	6.9	3,2	2,1	1,6	1,4	1,2	1,1	1.0	1,0
ude tor •9	70				1,1	1,3	1,5	1,9	2,6	4,5	18,7 3	4.8	3,5	2,2	1,7	1.4	1,2	1,1	1,0	1.0
Latitu ; = .	09			1,0	1,1	1,2	1.4	2,0	2,4	4 , 0	12,8 1	10,6	S, B	2,3	1.7	4.4	1,2	1,1	1.0	1.0
Error in Latitude for a Declination Augle of	5.0			1,0	1,1	1,2	1,4	1,7	2.3	3,7	6 6	13,8	4,1	512	1,8	4.4	1,2	1,1	1.0	1,0
Eri	4 0			1,0	1,1	1,2	4.	1,,	2,2	3,5	8,3	18,7	4.	2,6	1,8	1,5	1,3	1,1	1,1	1.0
u.	30			1,0	1,1	1,2	1,4	1.0	2.2	3,3	7,4	26,3 1	8.	2,7	1,3	1,5	1,3	1,1	1,1	1,9
.ube	20		1,0	1.0	1,1	1,2	1,3	1,6	2,1	3,2	8,8	37,4	5,0	2,7	1,9	1,5	1,3	1,1	1:1	1.0
IN LATITUBE	10		1,0	1,0	1,1	1,2	1,3	1,6	2,1	3,1	6,5	50,5	2,2	2,8	1,9	1,5	1,3	1,1	1,1	0
	0		1,0	1,0	<b>+</b> • •	1,2	1,3	1,0	2,1	3,1	4,4		2,2	2,8	1,9	1,5	1,3	+ + +	1,1	1,0
ERRCR	HA (+0R#)		ਜੰ	ਜ	ਜੰ	न	ਜ	न	8	ю	9	57,3	5	<b>N</b>	4		<del>г</del>	<b>ન</b>	н	++
		0.6	80	7.0	6.0	20	<b>A</b> 0	30	20	10	0	10	20	0 2 4	4.0	50	09.	0.20	980	06

						Err	or in ]	Table A6a Error in Longitude for a Declination Angle of $-18^{\circ}$	T; ude fo	Table A6a for a Decli	.6a :clinat:	ion An	igle of	-18°						
	ERROR	_ Z	LONGITUDE	TUDE		F 0 R	DEC	ີ ກ ເນ	118											
1 HA 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	HA (+0R=)	0	0	20	30	4	20	09	7.0	80	0.6	100	110	120	130	140	150	160	170	180
60																				
70	1,1		1,1	1,1																
60	1,1		1,1	1,1	f	1.1	1,1													
0	1.1		두 -	1,1	11	1.1	1,1	1,1												
0.4	1,1		1,1	1,1	1,1	1,1	1,1	1,1	1,1											
30	1,1		ન_	1,1	년 년	1.1	1,1	1,1	1,2											
20	1.1		£ 6 £	1.1	1,1	1,1	1,1	1,2	1,2	1.3										
10	1,1		1,1	1,1	۲ <b>،</b> ۲	1,2	1,3	7.4	4.5	4 8										
0	1,1		1,1	4,4	1,2	1.4	1,6	2,1	3,1	6,1	*									
10	1,4		1,1	4.	1,9	3,2	8,1	21,4	5,0	3.0	2,2									
20	1,1		1,7	2,3	2,3	2,2	2,0	α •	1,6	1,5	4 4									
30	1.1		1,1	1,1	1,2	1,2	£ 4	1,3	1,3	1,2	1,2	1,2								
0.4	1,1		1,1	1,1	1,1	1,1	1,1	1,1	۲.	1,1	1,1	1,1								
50	1,1		<b>~</b> !	1.1	स्त <u>ं</u> स्त	4.4	1,1	T + T	1,1	다. 다	F * F	t.	च । च							
j •	स	-1	다 다	1.1	ਜ ਜ	ਜ <b>਼</b>	1,1	1.	1,1	4.	1,1	다. 다	1,1	1,1						
7 C	T-1	н	rd "	1.1	다. 다	다.	1.1	ਰ ਰ	1,1	ਜ ਜ	1,1	다. 다	+. +.	다. 다	두:	1.1	ं <b>र</b> े स			
ت ت	vi	ᆏ	ज़्त ज़	ਜ <b>਼</b> ਜ਼	다. 러	ਜ <b>਼</b>	न्। न	ਦ <b>ੇ</b> ਦ	4.1	다.	다 다	1.	ਜ਼ ਜ	T • t	ਜ ਜ	11	∓: <del>-1</del>	1,1	F* F	1.1
ت ن <b>ن</b>	vi	14	. d	र-! • • -{	۳۱ ۲۱	<del>।</del> स	다 다	다 다	11	다. 다	-1	-! -!	4	년 대	ਜ ਵ	다 *!	क्र] • जर्म	1,1	4,4	1,1

Table A6b ror in Latitude for a Declination Angle of -18

	4 80																	1.0	4.4
	170																	1,0	1,1
	160																	1,0	1,1
	150																0.	1,0 1	1 H
	140																1.0	1,0 1	1,1
	130																1,0 1	1,0	# # # # # # # # # # # # # # # # # # #
18°	120															1,1	਼ ਜ•ਜ	1,0	1,1
Error in Latitude for a Declination Angle of -18° R DEC = -18	110														٤, ٤		 편 편	1.0	1.4
n Ang	100												1,9	1,5	۲. ان	1,2	ਜ ਜ	1, 1,	1,1
inatio	0 6									* * *	6,1	3,1	2,1	9 4	ं <b>क</b>	1,2	; ਜ! ਜ	: ਜ਼	4
a Decl	ВŲ							2,7	4.6	18,6 *	8,9	3,6	2,3	1.8	4.4	1,3	1,1	1,1	1,1
e for a	70					4.1	1,8	2,4	3,7	9,5 16	16,4 8	4,4	2,6 2	1,9 1	1,5 1	1,3 1	1,2 1	1,1	1,1
atitude = -18	69				2	4	1.6	2,1 2	3,2 3	6 49	77,016	5,6	2 6 5	2,0 1	1,6 1	1,3 1	1,2 1	1 1 1	1,1
r in L Dec	<b>5</b> 0			ન +	1.2	1,3	۲. د.	2.0.2	2,8	5,0 6	32,8 77	7,2 5	3,3 2	2,2 2	1,7 1	1,4,1	1,2 1	1,1	1 11
Erro FOR	40			ਾਂ ਜ਼	٠: وا وا	\$,1	v.	1.8	2,5	4,2	14,7 32	7 7.6	3,7 3	2,3	1,7 1	1.4	1,2 1	1,1	1.1
ū	<del>ي</del> د			E.	-1	7.4 7.4 7.4	4	4.7	2,3	3,7 4	0	Z.	eri.	Δ	30	4. H	1 2 1	<del>c</del> f	w.f
ne	02		. i	٠,١ ص	۳۱ ط دا	2,1	4.	4. 7.	2,2 2	3,4 3	8,3 10,	1 13,	4,5 4,	2,6 2,	1,8 1,	ارة 14	ار ا	ਜ ਜ	1 1,
) I I							4 ' I					19,1				1,5	1,3	<b>₹</b> • ₹	T .
N [. A	<b>+</b> f		 		•4	2.3	i.	ς: • 1	2,2	3,3	7,4	25,4	4,7	5,6	4 9	1,5	1,3	+ <del>-</del>	1,1
EFROR IN LATITUDE	9 ~ m≥		0 ° c	ر. ۲۰	त्त स	4	5.1.3	7.6	2,1	3,2	7,2	28,7	8 4	2,7	4,9	4	1,3	н Н	۲°
E	PA(+ <u>k</u> S#) PPJ 90	3 در	٤٤	نور	ر. طه	J. <b>7</b>	0.2	9	ĊŦ	0	610	0.50	930 930	440	9 87 0	<b>160</b>	a70	F 6 0	05=

Table A7a

						Er	Table A'a Error in Longitude for a Declination Angle of -27°	Longit	T ude fo	Table A7a for a Decli	r'/a :clinat	ion Ar	igle of	-27°						
m æ	ERROR	Z Z	LONGITU	TUDE		FOR	DEG	) II	-27											
HA (+0R#	~	0	10	20	30	4	5.0	60	7.0	80	06	100	110	120	130	140	150	160	170	• • •
1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0																				
80																				
7.0																				
09	1,1		1,1	1,1																
50	1,1		1,1	1,1	T o I	1,1	1,2													
0.4	1,1		1,1	1,1	111	1,2	1,2	1,2												
30	T - T		4,	1,1	1,2	1,2	1,2	1,3	1,3											
20	1,1		1,1	1,1	1,2	1,2	4 4 3	1,4	2,4											
07	1,1		1,1	1,2	1,2	1.3	4.4	1,6	1.9	2.4										
0	1,1		1,1	1,2	1,3	1.5	1,7	2,2	3,3	6,5	* *									
10	ਜ <b>਼</b> ਜ		1,2	1,3	1,6	2,1	3,1	4.0	* * *	6,5	4 4									
20	1+1		1,3	2,0	3,9	14.0	12,1	4.7	3,0	2,3	1,9	1,7								
30	τ•τ		1,7	2,3	2,4	2,3	2,1	1,9	1,7	1,6	1,5	1,4								
0 4	1,1		1,2	1,2	1,3	1,4	1,4	4 4	1,4	4,1	1,3	1,3	1,2							
000	T T		1,1	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2						
09:	1,1		1,1	1.1	1,1	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,1	1,1	1,1	1,1	٠		
.70	1,1		1,1	1,1	1,1	1,1	1,1	1,1	1.1	1,1	1,1	<b>ए</b> ।	1,1	1,1	T' =	1,1	1,1	1,1	1,1	• •
90	1,1		1,1	1,1	1,1	1,1	1,1	1,1	1,1	1.1	1,1	1,1	1,1	1.1	1,1	1,1	1.1	1,1	1,1	
0.5	ਜ	₩.	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	다. 다	1,1	1,1	1,1	1,1	

Table A7b Tror in Latitude for a Declination Angle of -27  $^{\circ}$ 

	170 180																1,0 1,0	1,0 1,0	1,1 1,1
	160																1,0 1	1,1	1,1
	150															1,0	1.0	1,1	1.
	140															1,1	1.0	1,1	1,1
	130															1,1	1,1	<b>T</b> • <del>T</del>	1,1
	120														1,2	1,1	1,1	1,1	1,1
	110													4.	1,3	1,2	4.4	1,1	1,1
	100											9,5	1,9	1.6	1.4	1,2	1.2	1,1	1,1
	06									* *	6,5	3,3	2,2	1,7	1,5	1,3	1,2	1,1	1,1
	80								4,3	12,7	13,0	A . 3	2.7	2.0	1,6	4.4	1,2	1,2	1,1
ì	70						1,7	2,2	3,3	4.	*	6,3	3,2	2,2	1,7	4.1	1,3	1,2	1,1
	0 9					1,3	1,6	1,9	2,6	4.	14,5	10,9	4,0	5,5	1 2 6	1,5	1,5	1,2	1,1
	50				11	1,3	4,4	1,7	2,3	3,4	7,5	32,8	5,2	2,9	2,0	1,6	1,4	1,2	1,1
5	4				1,1	1,2	1,3	1,6	2.0	2,9	5,3	45,3	6,9	3,3	2,2	1,7	1,4	1,2	1,1
	30				T t	4,4	1,3	2,1	1,8	2,5	4°	15,5	6,9	3,7	2,3	1,7	1,4	1,2	ਜ ਜ
<b>4</b> 00 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	20			1.0	1,0	1,1	1,2	4.4	1,7	2,3	3,8	10,4	13,1	4.1	2,4	1,8	1,4	1,2	1,1
	10			1,0	1,0	1,1	1,2	4.4	1,7	2,2	3,5	8,7	17,1	4,3	5,5	1,8	1,5	1,3	#
	HA(+0R=) 0			1,0	1,0	1,1	1,2	4.	1,7	2,2	3,4	8,2	19,1	4 4	2,6	8	4	1,3	ਜ <b>਼</b>
	PF 1	8 O	70	6.0	ις. O	0	30	20	10	9	010	020	020	640	950	09=	€70	0 8 0	06 8

## Appendix B

## EFFECT OF SYSTEMATIC ERRORS IN SHIP'S VELOCITY ON POSITION DETERMINATION

When the location of a moving ship is to be determined, it is necessary to include in the computation the doppler shift and position shift caused by the ship's motion relative to the moon. The error in the ship's velocity, if excessive, will be directly reflected in an error of position. However, by taking measurements over several hours, it is possible to reduce the errors in both position and velocity to the basic measurement error.

To show the required computation as well as the functional relation between position and velocity, the observation equation will be derived. The range and range rate including the ship's velocity are given as a function of time for an assumed stationary moon by

$$\rho(t) \approx \sqrt{s^2 + r^2 - 2 \, sr \, cos \, \beta(t)} + \int_{t_0}^{t} v_{sr} \, dt$$
 (B1)

$$\dot{\rho}(t) \approx -r \frac{d(\cos \beta)}{dt} + v_{sr} , \qquad (B2)$$

where  $v_{sr}$  is the radial velocity of the ship relative to the moon and the other symbols are as defined in Appendix A. For a short observation interval, the coordinates of the moon and the position of the ship are effectively stationary, and it is further assumed that the values of  $\rho$  and  $\dot{\rho}$  are referred to the midpoint of the observational interval  $T_0$ , or  $(t-t_0)/2 = T_0/2$ . Then

$$\rho_{T_0/2} \approx \sqrt{s^2 + r^2 - 2 \operatorname{sr} \cos \beta} + v_{sr} T_0/2$$
 (B3)

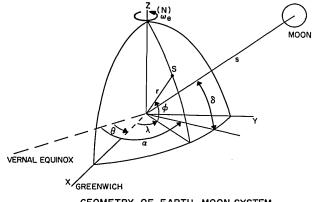
$$\dot{\rho}_{T_0/2} \approx -r \frac{d(\cos \beta)}{dt} \bigg|_{T_0/2} + v_{sr} . \tag{B4}$$

First,  $v_{sr}$  will be derived in Greenwich equatorial coordinates. Assume a coordinate system with the x axis pointing to the Greenwich meridian and the z axis to the north pole as shown in Fig. B1. Any point S on the earth's surface can be expressed in terms of the earth's radius r, the latitude  $\varphi$ , and the longitude  $\lambda$  ( $\lambda$  is defined as the negative of the conventional astronomical longitude).

The moon's position can be similarly described where the moon's distance is s, the latitude  $\varphi'$  is expressed as the declination  $\delta$ , and the longitude  $\lambda'$  is given by the difference between the right ascension  $\alpha$  and the Greenwich sidereal time  $\theta$ . Thus  $\lambda' = \alpha - \theta$ .

The radial velocity is obtained from

$$v_{sr} = -v_{s} \cdot \hat{\rho} \approx \frac{-v_{s} \cdot s}{s} . \tag{B5}$$



GEOMETRY OF EARTH-MOON SYSTEM

Fig. B1 - Geometry of the earth-moon system including celestial coordinates

where  $v_s$  is the ship's velocity vector, and the negative sign indicates that the radial velocity is measured relative to the center of the moon.

To determine the vector  $\mathbf{v}_s$  in equatorial coordinates, first define  $\mathbf{v}_s$  in a local coordinate system at the point of observation, with the y'' axis pointing east and the z'' axis pointing north. The x'' axis is normal to the y'' and z'' axes, and forms a right-handed coordinate system. The direction of  $v_s$  is measured by the angle  $\zeta$  from east to north as shown in Fig. B2.

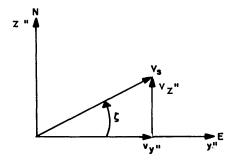


Fig. B2 - Components of ship's velocity

The components of  $v_s$  are then given by

$$v_{x''} = 0 ,$$
 
$$v_{y''} = v_s \cos \zeta ,$$
 
$$v_{z''} = v_s \sin \zeta .$$

To convert to the Greenwich equatorial coordinate system, two rotations are required. First rotate about the y'' axis by  $\varphi$ . This rotation brings the x'' axis parallel to the equatorial plane and the z'' axis parallel to the earth's axis of rotation. A second rotation by  $-\lambda$  about the z' axis brings the x' axis to the Greenwich meridian and makes the y' axis parallel to the y axis in Fig. B1. Using the transformation equation as given by Brouwer and Clemence,\* we obtain from the first rotation about y'' axis by  $\varphi$ 

$$v_{x'} = v_{x''} \cos \varphi - v_{z''} \sin \varphi = -v_{z''} \sin \varphi$$
,  $v_{y'} = v_{y''}$ ,  $v_{z'} = v_{x''} \sin \varphi + v_{z''} \cos \varphi = v_{z''} \cos \varphi$ ,

and from the second rotation about z' axis by  $-\lambda$ 

$$v_x = v_{x'} \cos \lambda - v_{y'} \sin \lambda ,$$

$$v_y = v_{x'} \sin \lambda + v_{y'} \cos \lambda ,$$

$$v_z = v_{z'} ,$$

giving finally the velocity vector in the Greenwich-equatorial coordinate system as

$$\mathbf{v}_{s} = -i \left( v_{y''} \sin \lambda + v_{z''} \sin \varphi \cos \lambda \right)$$

$$+ j \left( v_{y''} \cos \lambda - v_{z''} \sin \varphi \sin \lambda \right)$$

$$+ k v_{z''} \cos \varphi$$
(B6)

in terms of the east and north velocities  $v_{y''}$  and  $v_{z''}$ .

The lunar distance vector is

$$\mathbf{s} = \mathbf{s} \left[ i \cos (\alpha - \theta) \cos \delta + j \sin (\alpha - \theta) \cos \delta + k \sin \delta \right]. \tag{B7}$$

The velocity of the ship relative to the moon is then given by

$$v_{sr} = \frac{-\mathbf{v} \cdot \mathbf{s}}{s}$$

$$= \cos \delta \left[ \sin \gamma \ v_{v''} + \left( \sin \phi \cos \gamma - \tan \delta \cos \phi \right) \ v_{\tau''} \right], \tag{B8}$$

where  $\gamma$  is the local hour angle of the moon, i.e.,  $\gamma = \alpha - \theta - \lambda$  and  $\lambda$  is now defined by astronomical convention.

The range and range rate equations can then be expressed in Greenwich equatorial coordinates as

$$\rho \approx \sqrt{s^2 + r^2 - 2 \operatorname{sr} \cos \beta}$$

$$+ \frac{T_0}{2} \cos \delta \left[ \sin \gamma \ v_{y''} + \left( \sin \phi \cos \gamma - \tan \delta \cos \phi \right) \ v_{z''} \right]$$
(B9)

<sup>\*</sup>D. Brouwer and G. M. Clemence, "Methods of Celestial Mechanics," New York: Academic Press, p. 40, 1961.

and

$$\dot{\rho} \approx \cos \delta \left[ r \omega_e \cos \varphi \sin \gamma + \sin \gamma v_{y''} \right] + \left( \sin \varphi \cos \gamma - \tan \delta \cos \varphi \right) v_{z''}$$
 (B10)

The differential corrections are obtained from

$$\begin{split} \Delta \rho &= \frac{\partial \rho}{\partial \lambda} \, \Delta \lambda \; + \; \frac{\partial \rho}{\partial \phi} \, \Delta \phi \; + \; \frac{\partial \rho}{\partial v_{y''}} \, \Delta v_{y''} \; + \; \frac{\partial \rho}{\partial v_{z''}} \; \Delta v_{z''} \; \; , \\ \\ \Delta \dot{\rho} &= \frac{\partial \dot{\rho}}{\partial \lambda} \, \Delta \lambda \; + \; \frac{\partial \dot{\rho}}{\partial \phi} \, \Delta \phi \; + \; \frac{\partial \dot{\rho}}{\partial v_{y''}} \, \Delta v_{y''} \; + \; \frac{\partial \dot{\rho}}{\partial v_{z''}} \; \Delta v_{z''} \; \; . \end{split}$$

The eight coefficients are given to first-order accuracy (assume  $(T_0v_{y''})/2r$  and  $(T_0v_{z''})/2r << 1$  and  $(T_0/2) \Delta v_{y''}$ ,  $(T_0/2) \Delta v_{z''} < \sigma_m$ );

$$\frac{\partial \rho}{\partial \lambda} = -r \cos \phi \cos \delta \sin \gamma$$

$$\frac{\partial \rho}{\partial \psi_{y''}} = r \cos \delta \left( \sin \phi \cos \gamma - \cos \phi \tan \delta \right)$$

$$\frac{\partial \rho}{\partial v_{z''}} \approx 0$$

$$\frac{\partial \rho}{\partial \lambda} = -\cos \delta \left( r\omega_e \cos \phi \cos \gamma + \cos \gamma v_{y''} - \sin \phi \sin \gamma v_{z''} \right)$$

$$\frac{\partial \dot{\rho}}{\partial \lambda} = -\cos \delta \left( r\omega_e \sin \phi \sin \gamma - (\cos \phi \cos \gamma + \tan \delta \sin \phi) v_{z''} \right)$$

$$\frac{\partial \dot{\rho}}{\partial \psi_{y''}} = \sin \gamma \cos \delta$$

$$\frac{\partial \dot{\rho}}{\partial v_{y''}} = \sin \gamma \cos \delta$$

$$\frac{\partial \dot{\rho}}{\partial v_{y''}} = \cos \delta \left( \sin \phi \cos \gamma - \cos \phi \tan \delta \right) .$$

After converting to linear dimensions,

$$\Delta x = r \cos \varphi \Delta \lambda$$
,  
 $\Delta y = r \Delta \varphi$ ,

the observation equations become

$$\frac{\Delta \rho}{\cos \delta} = -\sin \gamma \, \Delta x + (\sin \phi \cos \gamma - \cos \phi \tan \delta) \, \Delta y , \qquad (B11)$$

$$\frac{\Delta \dot{\rho}}{\omega_{e} \cos \delta} = \left[ -\cos \gamma \left( \frac{v_{y''}}{r\omega_{e} \cos \phi} + 1 \right) + \sin \gamma \sin \phi \, \frac{v_{z''}}{r\omega_{e} \cos \phi} \right] \Delta x$$

$$+ \left[ -\sin \phi \sin \gamma + (\cos \phi \cos \gamma + \tan \delta \sin \phi) \, \frac{v_{z''}}{r\omega_{e}} \right] \Delta y$$

$$+ \sin \gamma \, \frac{\Delta v_{y''}}{\omega_{e}} + (\sin \phi \cos \gamma - \cos \phi \tan \delta) \, \frac{\Delta v_{z''}}{\omega_{e}} . \qquad (B12)$$

For the moderate velocities of ships  $v_y$  "/( $r\omega_e \cos \varphi$ ),  $v_z$  "/( $r\omega_e \cos \varphi$ ), and  $v_z$  "/( $r\omega_e$ ) << 1, and by rearranging the equations for solution of  $\Delta x$  and  $\Delta y$ , we obtain

$$\frac{\Delta \rho}{\cos \delta} = -\sin \gamma \, \Delta x + (\cos \gamma \, \sin \varphi - \cos \varphi \, \tan \delta) \, \Delta y \tag{B13}$$

and

$$\frac{\Delta \dot{\rho}}{\omega_{e} \cos \delta} - \sin \gamma \frac{\Delta v_{y''}}{\omega_{e}} - (\sin \varphi \cos \gamma - \cos \varphi \tan \delta) \frac{\Delta v_{z''}}{\omega_{e}}$$

$$= -\cos \gamma \Delta x - \sin \gamma \sin \varphi \Delta y . \tag{B14}$$

Let

$$\begin{split} m_1 &= \frac{\Delta \rho}{\cos \delta} \;; \\ m_2' &= \frac{\Delta \dot{\rho}}{\omega_e \cos \delta} - \sin \gamma \, \frac{\Delta v_y''}{\omega_e} - (\cos \gamma \, \sin \phi - \cos \phi \, \tan \delta) \, \frac{\Delta v_z''}{\omega_e} \;, \end{split}$$

and

$$a_{11} = -\sin \gamma ,$$

$$a_{12} = \cos \gamma \sin \varphi - \cos \varphi \tan \delta ,$$

$$a_{21} = -\cos \gamma ,$$

$$a_{22} = -\sin \gamma \sin \varphi ;$$

then

$$\Delta x = \frac{\begin{vmatrix} m_1 & a_{12} \\ m'_2 & a_{22} \end{vmatrix}}{\Delta} = \frac{m_1 a_{22} - m'_2 a_{12}}{\Delta},$$

$$\Delta y = \frac{\begin{vmatrix} a_{11} & m_1 \\ a_{21} & m'_2 \end{vmatrix}}{\Delta} = \frac{m'_2 \ a_{11} - m_1 \ a_{21}}{\Delta} ,$$

where  $\Delta = a_{11}a_{22} - a_{12}a_{21} = \sin \varphi - \cos \gamma \cos \varphi \tan \delta$ , or

$$\Delta x = \frac{1}{\Delta} \left( -\sin y \sin \varphi \frac{\Delta \rho}{\cos \delta} - a_{12} \frac{\Delta \dot{\rho}}{\omega_e \cos \delta} + a_{12} \sin y \frac{\Delta v_{y''}}{\omega_e} + a_{12}^2 \frac{\Delta v_{z''}}{\omega_e} \right), \tag{B15}$$

$$\Delta y = \frac{1}{\Delta} \left( -\sin \gamma \frac{\Delta \dot{\rho}}{\omega_e \cos \delta} + \sin^2 \gamma \frac{\Delta v_{\gamma''}}{\omega_e} \right)$$

$$+ \sin \gamma \ a_{12} \frac{\Delta v_{z''}}{\omega_e} + \cos \gamma \ \frac{\Delta \rho}{\cos \delta}$$
 (B16)

The dependence of the variance of  $\Delta x$  and  $\Delta y$  on the variance of  $\Delta v_{y''}$  and  $\Delta v_{z''}$  is found from

$$\sigma_{\Delta x_{v}}^{2} = \left\{ \begin{bmatrix} \frac{\partial \Delta x}{\partial v_{y''}} & \frac{\sigma_{\Delta v_{y''}}^{2}}{\omega_{e}} + \frac{\partial \Delta x}{\partial v_{e}} & \frac{\sigma_{\Delta v_{z''}}^{2}}{\omega_{e}} \\ \frac{\partial \Delta x}{\partial v_{e}} & \frac{\partial \Delta x}{\partial v_{e}} & \frac{\sigma_{\Delta v_{z''}}^{2}}{\omega_{e}} \end{bmatrix} \right\}$$

and

$$\sigma_{\Delta y_{v}}^{2} = \left\{ \left[ \frac{\partial \Delta y}{\left( \frac{\Delta y_{y''}}{\omega_{e}} \right)} \right]^{2} \frac{\sigma_{\Delta v_{y}}^{2}}{\omega_{e}} + \left[ \frac{\partial \Delta y}{\left( \frac{\Delta v_{z''}}{\omega_{e}} \right)} \right]^{2} \frac{\sigma_{\Delta v_{z''}}^{2}}{\omega_{e}} \right\}$$

Letting

$$\frac{\sigma_{\Delta v_{y''}}^2}{\omega_e} = \frac{\sigma_{\Delta v_{z''}}^2}{\omega_e} = \sigma_m$$

and substituting for

$$\frac{\partial \Delta x}{\partial \left(\frac{\Delta v_{y''}}{\omega_e}\right)} = \frac{1}{\Delta} \; a_{12} \; \sin \; \gamma \; , \qquad \frac{\partial \Delta x}{\partial \left(\frac{\Delta v_{z''}}{\omega_e}\right)} = \frac{1}{\Delta} \; a_{12}^2 \; ,$$

$$\frac{\partial \Delta y}{\partial \left(\frac{\Delta v_{y''}}{\omega_e}\right)} = \frac{1}{\Delta} \sin^2 \gamma , \quad \frac{\partial \Delta y}{\partial \left(\frac{\Delta v_{z''}}{\omega_e}\right)} = \frac{\sin \gamma \ a_{12}}{\Delta} .$$

the standard deviations of  $\Delta x$  and  $\Delta y$  are given by

$$\sigma_{\Delta x_{y}} = \frac{1}{\Lambda} \left( a_{12}^{2} \sin^{2} \gamma + a_{12}^{4} \right)^{1/2}$$
 (B17)

and

$$\sigma_{\Delta y_{v}} = \frac{1}{\Delta} \left( \sin^{4} \gamma + a_{12}^{2} \sin^{2} \gamma \right)^{1/2}$$
 (B18)

The effective errors in position based on measurement errors were obtained in Appendix A as

$$\sigma_{\Delta x_m} = \frac{1}{\Delta} \left( a_{22}^2 + a_{12}^2 \right)^{1/2} \sigma_m$$
,

and

$$\sigma_{\Delta y_m} = \frac{1}{\Delta} \sigma_m$$
.

The ratios  $\sigma_{\Delta x_{v}}/\sigma_{\Delta x_{m}}$  and  $\sigma_{\Delta y_{v}}/\sigma_{\Delta y_{m}}$  then express the increase in the error of the position due to the velocity error.

These ratios, expressed as

$$R_{1} = \frac{\sigma_{\Delta x_{v}}}{\sigma_{\Delta x_{m}}} = \frac{a_{12} \left(\sin^{2} \gamma + a_{12}^{2}\right)^{1/2}}{\left(a_{22}^{2} + a_{12}^{2}\right)^{1/2}}$$
(B19)

and

$$R_2 = \frac{\sigma_{\Delta y_v}}{\sigma_{\Delta y_m}} = \sin y \left(\sin^2 y + a_{12}^2\right)^{1/2}$$
, (B20)

are computed as a function of time and position and presented in Tables B1 through B7. For velocity errors that are large relative to the measurement errors, the numbers of the table are multiplied by  $(\sigma_{\Delta v}/\omega_e)/\sigma_m$  to indicate the normalized position error introduced by the velocity error. The computer printout is the program used to solve Eqs. (B19) and (B20).

## Computer Program Used to Solve Eqs. (B19) and (B20)

TN5.4A 09/09/68

```
PROGRAM COVERAGE
    DIMENSION SIGX(19,7,19), SIGY(19,7,19), IPRINT(3)
    DIMENSION
                R1(19,7,19),R2(19,/,19),R3(19,7,19),R4(19,7,19)
    COMMON/1/SIGX,R1,R2
    COMMON/2/SIGY,R3,R4
    TYPE INTEGER DCLN, HA
    TYPE DOUBLE
                   DEC, PHI, GAMMA, TAND, SINP, COSP, COSG, DELTA, COSD, SIND
    TYPE DOUBLE
                  NUMERATR
    TYPE DOUBLE
                  SING
    DATA (IPRINT = 8H(***,5X,,0,6HF5.3))
900 FORMAT (+1
                       ERROR IN LONGITUDE
                                                FOR
                                                         DEC =*,14,/)
                                     15
901 FORMAT (+0+,5x,+HA(+GR-) 0
                                            30
                                                  45
                                                               75
                                                         60
               135
   *105
          120
                              165
                       150
                                    180+,/,+
                                                PHI +,/)
902 FORMAT (*+*, 15)
903 FORMAT (+1
                       ERROR IN LATITUDE
                                                F0R
                                                         DEC =+, [4,/)
904 FORMAT (+0+)
905 FORMAT (79H1
                        R1 = A12*((A12**2 + SING**2)/(A22**2 + A12**2))
                       DEC = , [4,/)
R2 = SING*(A12**2 + SING**2)**1/2
   ***1/2
               FOR
906 FORMAT (60H1
                                                                 FOR
   *EC = ,14,/)
907 FORMAT (#1
                       EFFECT OF V SUB Z ON DELTA Y
                                                        FOR
                                                              DEC =+,[4,/)
908 FORMAT (#1
                       EFFECT OF V SUB Y ON DELTA Y
                                                              DEC =+, [4,/)
                                                        FOR
110 FORMAT (*5X,*,12,*(1X*)
    DTR = 0,017453292519943296D
             J=1,7
    DO 100
    DCLN = 9*(1-J) + 27
    DEC = DCLN+DTR
    COSD = DCOS(DEC)
    SIND = DSIN(DEC)
    TAND = SIND/COSD
    DO 50
            I = 1,19
    LAT = 10*(1 - I) + 90
    PHI = LAT*DTR
    SINP = DSIN(PHI)
   COSP = DCOS(PHI)
   no 30
           K = 1,15
   HA = 15*(K - 1)
   GAMMA = HA*DTR
   COSG = DCOS(GAMMA)
   SING = DSIN(GAMMA)
   A12 = ABSF(COSG*SINP - COSP*TAND)
   A22 = -SINP+SING
   IF(SINP+SIND + COSP+COSD+COSG GT. 0) GO TO 20
   SIGX(I,J,K) = SIGY(I,J,K) = R^{1}(I,J,K) = R^{2}(I,J,K) = R^{3}(I,J,K)
  * R4(I,J,K) = 0.
   GO TO 30
20 DELTA = SINP - COSP+COSG+TAND
   IF(DELTA .EQ. 0)    DELTA = 1,0D+30
R1(I;J;K) = A12*SQRTF(A12**2 + SING**2)/SQRTF(A22**2 + A12**2)
   R2(I_JJ_JK) = SING + SQRTF(A12 + + 2 + SING + + 2)
   SIGX(I,J,K) = SQRTF(1, + R1(I,J,K)**2)
   SIGY(I,J,K) = SQRTF(1. + R2(I,J,K)**2)
30 CONTINUE
50 CONTINUE
   PRINT 905,
                 DCLN
   PRINT 901
```

D

TN5,4A

09/09/68

```
D0135 I = 1,19
   LAT = 10*(1 - I) + 90
   PRINT 902, LAT
   M = 0
    DO 133 L=1,13
    K = 14 - L
    IF(R1(I,J,K) .NE. 0.) G0 T0 134
133 K = K - 1
134 M = K
    ENCODE (8,110, IPRINT(2)) M
PRINT IPRINT,
135 PRINT 904
                      (R1 (I,J,K),K=1,M)
    PRINT 906,
               DÇLN
    PRINT 901
    D0235
          I = 1.19
   LAT = 10*(1 - I) + 90
   PRINT 902, LAT
   M = 0
   DO 233 L=1,13
    K = 14 - L
    IF(R2(I,J,K) .NE. 0,) G0 T0 234
233 K = K - 1
234 M * K
   ENCODE (8,110, IPRINT(2)) M
                      (R2 (I,J,K),K=1,M)
   PRINT IPRINT,
235 PRINT 904
   PRINT 900, DCLN
    PRINT 901
    DO 35
           I = 1,19
   LAT = 10*(1 - I) + 90
   PRINT 902, LAT
   M = 0
    DO 33 K=1,13
    IF(SIGX(I,J,K) .EQ. 0.) G0 T0 34
 33 M = M + 1
34 IF(M .EQ. 0) G0 T0 35
    ENCODE (8,110, IPRINT(2)) M
                   (SIGX(I,J,K),K=1,M)
   PRINT IPRINT,
 35 PRINT 904
   PRINT 903, DCLN
PRINT 901
   D0 40 I = 1,19
   LAT = 10*(1 - I) + 90
   PRINT 902, LAT
   M = 0
    DO 38 K=1,13
    IF(SIGY(I,J,K) .EQ. 0.) G0 T0 39
 38 M = M + 1
 39 IF(M .EQ, 0) G0 T0 40
    ENCODE (8,110, IPRINT(2)) M
                     (SIGY(I,J,K),K=1,M)
    PRINT IPRINT,
 40 PRINT 904
100 CONTINUE
    END
```

	Pos	ition E	rror in	Table B1a Position Error in the $\times$ Coordinate Caused by the Velocity Error for	oordin	Table B1a te Caused I	B1a sed by	the Velo	ocity E	ror for	. 8 = 27°	o	
	R1 = A12+(	((A12*	*2 + S	A12*((A12**2 + SING**2)/(A22**2 + A12**2))**1/2	/(A22	**2 + 1	412**2	)+#1/5		FOR	DEC =	. 27	
<u> </u>	HA(+0R-) 0	15	30	4. IV	9	75	06	105	120	135	150	165	180
06	1,000		0,966 0,866	0,707 0,500 0,259	0.500	0,259		0,000 1,259	0.500	0,707	0.866	996'0	1,000
80	968'0		0.864 0.768	0,613	0.409	0.169	060.0	0,613 0,409 0,169 0,090 0,348	0,587	0.790	0.944	1,041	1.073
70	0,765	0,738	0.654	0,654 0,511 0,312 0,073	0,312	0,073	0,185	0,185,440 0,670	0,670		0,860 1,000	1,085	1,114
6.0	0,611	0,594	0.530	0,594 0,530 0,400 0,204 0,035 0,291 ,536	0,204	0.035	0,291	,536	0,747 0,914 1,031	0,914	1.031		
20	0,439	0,439		0,397 0,272	0,072	0.072 0.168	0,414	.637	0,819				
40	0,252	0,281	0.242	0.099	0,107	0,336	0,557	1.,744					
30	0,059	0.080	0.016	0.172	0,358	0,551	0,723	856					
20	0,137	0,256	0,389	0,522	0,662	0,662 0,795	0.902						
10	0,328		0,593	0,419 0,593 0,763 0,903 1,003	0,903	1,005	1.057						
0	0,510	0.571	0.714	0.571 0.714 0.872 1.005 1.092	1,005	1,092	1,122						
-10	0,675	0.716	0,815	0.926	1,015	1,061							
-20	0,821	0.845	0,901	0.957	0,985	996.0					•		
0 87	0,941	0.951	0,968	0,972	0,939								
4.0	1,033	1.030	1.014	0.970	0,883								
1 0	1,094	1.080	1,094 1.080 1,035	0.951									
160	1,121	1,121 1.099											

	27
	8 = 27
	for
	ty Error
	Veloci
	the
_	l by
Table B1b	Caused
	Coordinate
	×
	ı the
	in
	Error
	Position

	Pos	ition E	Table B1b Position Error in the $_{\it y}$ Coordinate Caused by the Velocity Error for	the $_{Y}$ C	oordina	Table B1b te Caused	B1b sed by t	he Velo	ocity Er	ror for	δ = 27°	0	
R2	= SING	SING+(A12++2	+	SING*#2)*#1/2	(**1/2	F.	FOR	DEC =	27				
HA(+0R-) PhI	٥-2	15	30	4.00	09	15	06	105	120	135	150	165	180
06	00000	0.259	0.500	0.707	0,866	996.0	1,000	996.	998.0	0.707	0.500	0,259	000'0
0 8	0.000	0.233	0.457	0.659	0.828	0,947	1.004	066	0,903	0,747	0.533	0,277	000.0
7.0	00000	0,201	0.406	0.608	0,792	935	1,015	,016	0,935	0,776	0.554	0,288	0.00.0
0.9	00000	0.165	0,352	0.560	0,766	0,935	1,032	.041	0,958	0,791	0.561		
ю 0	000.0	0,126	0.301	0,522	0,752	0.941	1,052	1,062	0.970				
0.4	000'0	0.090	0.263	0.502	0,752	0,958	1.073	.077					
30	00000	0.068	0,250	0.504	0,768	0.980	1,093	.084					
20	000.0	0.077	0.266	0.527	0,796	1.006	1,109						
10	000'0	0,109	0.306	0.567	0,832	1.032	1,119						
0	00000	0.148	0,357	0,616	0.870	1,055	1,122						
-10	00000	0.186	0.411	0,667	0,907	1,072							
-20	000,0	0.220	0.461	0.714	0,938	1,082							
430	00000	0,248	0.504	0,752	096.0								
140	000'0	0.270	0,535	0,779	0,971								
50	00010	0.284	0,555	0,792									
, 0 9 -	000.0	0.290	_										
-70													
0			,										
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	$\delta = 18^{\circ}$
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able B2a	Caused b
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	$\mathbf{r}$ in the $\mathbf{x}$ C
	Error i
	osition

Ρc	Position Error in the $\times$ Coordinate Caused by the Velocity Error for $\delta=18^{\circ}$	rror in	the x C	oordina	te Cau	sed by tl	ne Velo	city E	ror for	<i>β</i> = 18	0	
	R1 = A12*((A12**2 + SING**2)/(A22**2 + A12**2))**1/2	*2 + S	ING**2	/(A22	**2 + 1	A12**2)	)***(		FOR	DEC =	18	
_	HA(+@R-) 0 15	15 30 45 60 75	45	9	75	06	105	120	105 120 135	150	165	180
=	1,000 0,966 0,866 0,707 0,500 0,259 0,000	0.866	0.707	0.500	0,259	000.0	,259	0,500	.259 0,500 0,707 0,866 0,966 1.000	0.866	996.0	1.000
~	0,928 0,896 0,800 0,645 0,441 0,201 0,057	0.800	0.645	0.441	0.201	0.057		0,555	.316 0,555 0,758 0,912 1,009 1.041	0.912	1,009	1.041
Ñ	0,829 0,801 0,717 0,575 0,378 0,140 0,118	0.717	0.575	0,378	0.140	0,118	.374	0,606	.374 0,606 0,797 0,937	0.937		
ò	0,704 0,685 0,621 0,496 0,308 0,071 0,187	0.621	0.496	0.308	0.071	0,187	436	,436 0,654				
iù	0,557 0.553 0.517 0.409 0.224 0.014 0.269	0.517	0.409	0.224	0.014	0,269	.505					
6	0,394 0,414 0,406 0,304 0,112 0,128 0,372	0.406	0.304	0,112	0.128	0,372	585					
7	0,219 0,276 0,271 0,142 0,063 0,294 0,509	0.271	0.142	0.063	0,294	0.509						
3	0,037 0,071 0,027 0,180 0,362 0,543 0,696	0.027	0.180	0,362	0,543	969.0						
4	0,146 0.288 0,470 0,623 0,754 0,857 0,923	0.470	0,623	0,754	0,857	0.923						
2	0,325 0,415 0,596 0,778 0,925 1,019 1,051	0.596	0,778	0,925	1,019	1,051						
6	0,494 0,550 0,675 0,804 0,897 0,938	0,675	0.804	0,897	0,938							

0,781 0,796 0,823 0,830 0,788 0,680 0,975 0,963 0,921 0.836 0,698 0,892 0.891 0.881 0.838 0,742

> 140 150

0,647 0,680 0,752 0,818 0,839 0,799

-20 330 1,028 1,007 0,940 0,823 160

1,051 1.022

-70 -60 -90

	for $\delta = 18^{\circ}$
	Error f
	Velocity
<u>م</u>	by the
<b>Fable B2b</b>	Caused
H	Coordinate
	in the $_{y}$
	osition Error
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	Pos	ition Er	Table B2b Position Error in the $_{\it y}$ Coordinate Caused by the Velocity Error for	the y Cc	, ordinat	Table B2b te Caused	s2b ed by tl	he Velo	city Erı	cor for	$\delta = 18^{\circ}$		
R2 :	S SING	*(A12*	SING*(A12**2 + SING**2)**1/2	ING**2	)**1/2	Ĭ.	FOR	DEC =	18				
HA(+0R-) PFI	0	15	30		•	15	06	105	120	135	150	165	180
0.5	000.0	0.259		0.500 0.707 0.866 0.966 1.000 0.966 0.866 0.707	0.866	996'0	1,000	996.0	0,866	0,707	0,500	0,259	0.000
80	000.0	0.241	0.470	0.674	0,840	0,953	1.002	0.980	0,888	0.730	0.519	0,269	0.000
7.0	000.0	0.217	0,431	0,635	0,812	0,942	1,006	0,942 1,006 0,994	0,903	0,742	0.526		
60	000*0	0.187	0,386	0,386 0,593	0,786	935	1,013	0,786 0,935 1,013 1,005	0,910				
0.00	000.0	0.153	0,000 0.153 0.338 0.553 0,765 0,943 1,022 ',013	0.553	0,765	0,933	1,022	1,013					
4 0	00000	0.117	0,294	0,294 0,521	0,753	0,936	1,031	3,016					
30	000'0	0.085	0.261	0,503	0.750	0,944	1,039						
20	000.0	0.067	0.250	0.502	0,759	956.0	1.046						
10	00000	0,000 0,078	0.264	0.264 0.519 0.777	0,777	0/6.0	1,050						
0	00000	0.108	0,108 0,298	0.550	0.550 0.801	0.984	1,051						
-10	00000	0.143	0.343	0,590	0,829	0,997							
-20	00000	0.178	0,391	0,632	0,856	1,008							
02-	00000	0,000 0,209	0.436	0.672	0.672 0,880	1,014							
140	000'0	0,235		0.474 0.705 0.898	0.898					4			
-50	000'0	0.255	0,503	0.729	0,908	٠		^			•		,
-60	000 0	0.267	0.520	0.742									
-70	000'0	0.272											
0													

	Po	sition E	Table B3a Position Error in the × Coordinate Caused by the Velocity Error for	the x C	Coordin	Table B3a ate Caused	B3a sed by	the Vel	ocity E	rror for	δ = 9°		
	R1 = A12*	A12*((A12**2	1*2 + S	+ SING*#2)/(A22**2	)/(A22	**2 +	+ A12**2))**1/2	))**1/	8	F 9 R	DEC *	0.	
i i	HA(+@R-) 0	15	30	45	9	75	0.6	105	120	135	150	165	18
0.6	1,000		0.966 0.866 0.707 0,500 0,259 0,000 0,259 0,500 0,707 0.866 0,966 1.00	0.707	0,500	0,259	0.000	. 259	0.500	0,707	0.866	996.0	1.00
80	0,957	0.925	0.829	0.674	0.470	0,231	0,028		1,286 0,526	0,729	0.884		
7.0	988.0	0.858	0,773	0.632	0,437	0.201	0,058	0,315					
9	0,787	0.767		0,703 0,582	0,399	0.167	0,091	1, 345					
الا 0	0,664		0,658.0,621.0,525	0,525	0,355	0.125	0.132						
40	0,521	0.534	0.533	0.462	0,301	0.0.0	0.187						
30	0,363	0,405	0.444	0.386	0,220	0.016	0,267						
0.7	0,193		0,284 0,340	0,256	0,065	0.174	0.403						
10	0,018		0,066 0,032	0.185	0,363	0,537	0.676						
0	0.158	0.303	0.524	0,725	0.880	6/6.0	1.012						
-10	0,330	0.411	0.564	969.0	0,765	0,757							
-20	0,491	0.536	0,625	0.687	0,677	0.580							
-30	0,637	0.658	0.695	0.698	0,632	0.484							
9	0,764	0.767	0.761	0.716	0,605	0,424							
-50	0,868	0,857	0,817	0.731	0,586	0.380							
09-	0.0	0,924	0.858	0.740	0,567								
-70	0,994	0.966	0.881	0.740	0.548								
00	1,012	0.980											

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Table B3b	Caused
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HAIL-MAIL-MAIL-MAIL-MAIL-MAIL-MAIL-MAIL-M		Po	Table B3b Position Error in the $y$ Coordinate Caused by the Velocity Error for	rror ii	$1  ext{the } y$ (	Coordin	Table B3b ate Caused	B3b ised by	the Vel	ocity E	rror fo	r 8 = 9°		
0,000 0.259 0.500 0.707 0.866 0.965 1.000 0.966 0.707 0.500 0.259 0.000 0.259 0.203	œ	*	i+(A12+		ING*#2	)**1/2		<b>1</b> 2		•				
0,000 0.259 0.500 0.707 0.866 0.966 1.000 0.966 0.707 0.500 0.259 0.000 0.248 0.483 0.688 0.851 0.959 1.000 0.972 0.875 0.716 0.506 0.000 0.231 0.455 0.660 0.832 0.951 1.001 0.976 0.000 0.231 0.455 0.660 0.832 0.951 1.001 0.976 0.000 0.231 0.455 0.660 0.832 0.951 1.001 0.976 0.000 0.178 0.535 0.700 0.944 1.007 0.909 0.12 0.290 0.523 0.770 0.944 1.007 0.000 0.112 0.290 0.523 0.776 0.933 1.009 0.000 0.112 0.290 0.523 0.756 0.935 1.012 0.000 0.007 0.207 0.500 0.501 0.752 0.935 1.012 0.000 0.007 0.293 0.501 0.752 0.945 1.012 0.000 0.107 0.293 0.571 0.800 0.961 0.002 0.007 0.293 0.571 0.800 0.961 0.007 0.203 0.411 0.335 0.571 0.800 0.961 0.000 0.114 0.335 0.571 0.800 0.901 0.901 0.000 0.228 0.457 0.676 0.860 0.977 0.000 0.228 0.457 0.676 0.860 0.977 0.000 0.228 0.457 0.676 0.860 0.977 0.000 0.228 0.457 0.678 0.871 0.000 0.258 0.501 0.713 0.877	HA						15		105			150	165	180
0,000 0,259 0,500 0,707 0,866 0,966 1,000 0,966 0,707 0,500 0,259 0,000 0,248 0,485 0,688 0,881 0,959 1,000 0,972 0,875 0,716 0,506 0,000 0,231 0,455 0,660 0,832 0,951 1,001 0,976 0,000 0,207 0,418 0,626 0,810 0,945 1,003 0,978 0,000 0,178 0,331 0,553 0,770 0,945 1,007 0,000 0,112 0,290 0,523 0,770 0,944 1,007 0,000 0,112 0,290 0,523 0,770 0,944 1,007 0,000 0,007 0,262 0,512 0,750 0,945 1,012 0,000 0,007 0,262 0,512 0,750 0,945 1,012 0,000 0,007 0,262 0,512 0,752 0,945 1,012 0,000 0,007 0,262 0,512 0,762 0,945 1,012 0,000 0,114 0,335 0,571 0,800 0,941 0,012 0,900 0,114 0,335 0,571 0,800 0,941 0,014 0,379 0,609 0,842 0,974 0,000 0,228 0,457 0,676 0,800 0,977 0,000 0,228 0,484 0,699 0,871 0,000 0,258 0,501 0,713 0,877 0,000 0,258 0,501 0,713 0,877 0,000 0,258 0,501 0,713 0,877 0,000 0,258 0,501 0,713 0,877 0,000 0,258 0,501 0,713 0,877 0,000 0,258 0,501 0,713 0,877	T T													
0,000 0,234 0,485 0,688 0,851 0,959 1,000 0,972 0,875 0,716 0,000 0,231 0,455 0,660 0,832 0,951 1,001 0,976 0,000 0,207 0,418 0,626 0,810 0,945 1,003 0,978 0,000 0,178 0,376 0,589 0,789 0,938 1,005 0,000 0,146 0,331 0,553 0,776 0,934 1,007 0,000 0,146 0,321 0,553 0,776 0,934 1,007 0,000 0,112 0,290 0,523 0,756 0,933 1,009 0,000 0,002 0,261 0,504 0,750 0,935 1,011 0,000 0,007 0,262 0,512 0,762 0,935 1,012 0,000 0,107 0,293 0,572 0,939 1,012 0,000 0,107 0,293 0,572 0,945 1,012 0,000 0,174 0,335 0,571 0,800 0,961 0,000 0,174 0,335 0,571 0,800 0,961 0,000 0,228 0,457 0,645 0,842 0,974 0,000 0,228 0,457 0,649 0,842 0,974 0,000 0,258 0,501 0,713 0,877 0,000 0,258 0,501 0,713 0,877 0,000 0,268 0,269 0,871 0,000 0,268 0,269 0,871	06	00000	0.259	0.500	0.707	998'0		1,000	996.	0,866		0.500	0,259	0.000
0,000 0,231 0,455 0,660 0,832 0,000 0,207 0,418 0,626 0,810 0,000 0,178 0,376 0,589 0,789 0,000 0,000 0,112 0,290 0,523 0,770 0,000 0,082 0,261 0,504 0,750 0,000 0,0079 0,262 0,512 0,762 0,000 0,000 0,107 0,293 0,512 0,762 0,000 0,107 0,293 0,512 0,762 0,000 0,174 0,379 0,609 0,872 0,000 0,228 0,457 0,679 0,871 0,000 0,228 0,484 0,699 0,871 0,000 0,258 0,501 0,713 0,877 0,000 0,262	80	0000						1,000						
0,000 0.207 0.418 0.626 0.810 0.000 0.178 0.376 0.589 0.789 0.000 0.146 0.331 0.553 0.770 0.000 0.112 0.290 0.523 0.755 0.000 0.000 0.000 0.000 0.250 0.261 0.504 0.755 0.000 0.000 0.0079 0.262 0.512 0.762 0.000 0.107 0.293 0.537 0.779 0.000 0.174 0.335 0.571 0.800 0.000 0.203 0.421 0.649 0.822 0.000 0.228 0.457 0.679 0.860 0.000 0.258 0.464 0.699 0.871 0.000 0.258 0.501 0.713 0.877 0.000 0.262	20	00000		0.455	0,660			1,001						
0,000 0.178 0.376 0.589 0,789 0.938 0,000 0.146 0.331 0.553 0,770 0,934 0,000 0.112 0.290 0.523 0,756 0.933 0,000 0.012 0.261 0.564 0.750 0.935 0,000 0.067 0.250 0.501 0.752 0.939 0,000 0.107 0.293 0.512 0.762 0.945 0,000 0.107 0.293 0.537 0.779 0.945 0,000 0.174 0.335 0.571 0.800 0.961 0,000 0.203 0.421 0.645 0.842 0.968 0,000 0.228 0.457 0.676 0.860 0.977 0,000 0.258 0.501 0.713 0.871 0,000 0.258 0.501 0.713 0.871	09	0000	0.207	0,418	0.626			1,003	. 9.978					
0,000 0.146 0.331 0.553 0.770 0.934 0.000 0.112 0.290 0.523 0.756 0.933 0.000 0.082 0.261 0.504 0.750 0.935 0.000 0.067 0.250 0.501 0.752 0.935 0.000 0.067 0.250 0.501 0.752 0.935 0.000 0.107 0.293 0.512 0.762 0.945 0.000 0.174 0.379 0.537 0.779 0.963 0.000 0.174 0.379 0.609 0.822 0.968 0.000 0.203 0.421 0.645 0.842 0.974 0.000 0.228 0.457 0.676 0.860 0.977 0.000 0.246 0.484 0.699 0.871 0.000 0.262	130	00000				0,789	0,938	1,005						
0,000 0.112 0.290 0.523 0.756 0.933 0,000 0.082 0.261 0.504 0.750 0.935 0,000 0.067 0.250 0.501 0.752 0.939 0,000 0.067 0.250 0.501 0.752 0.945 0,000 0.107 0.293 0.537 0.779 0.945 0,000 0.174 0.335 0.571 0.800 0.961 0,000 0.203 0.421 0.645 0.842 0.974 0,000 0.228 0.457 0.676 0.860 0.977 0,000 0.246 0.484 0.699 0.871 0.000 0.258 0.501 0.713 0.871 0.000 0.262	0	0000		0,331				1,007						
0,000 0.082 0.261 0.504 0.750 0.935 0.000 0.067 0.250 0.501 0.752 0.939 0.000 0.079 0.262 0.512 0.762 0.945 0.000 0.107 0.293 0.537 0.779 0.953 0.000 0.174 0.335 0.571 0.800 0.961 0.000 0.203 0.421 0.649 0.842 0.974 0.000 0.228 0.457 0.679 0.860 0.977 0.000 0.246 0.484 0.699 0.871 0.000 0.262	30	00000			0,523			1,009						
0,000 0.067 0.250 0.501 0.752 0.939 0.000 0.079 0.262 0.512 0.762 0.945 0.000 0.107 0.293 0.537 0.779 0.953 0.000 0.141 0.335 0.571 0.800 0.961 0.000 0.174 0.379 0.609 0.822 0.968 0.000 0.203 0.421 0.645 0.842 0.974 0.000 0.228 0.457 0.676 0.860 0.977 0.000 0.258 0.501 0.713 0.871 0.000 0.262	50	0000			0.504			1.011						
0,000 0.079 0.262 0.512 0.762 0.945 0.000 0.107 0.293 0.537 0.779 0.953 0.000 0.141 0.335 0.571 0.800 0.961 0.000 0.174 0.379 0.609 0.822 0.968 0.000 0.203 0.421 0.645 0.842 0.974 0.000 0.228 0.457 0.676 0.860 0.977 0.000 0.258 0.501 0.713 0.871 0.000 0.262	10	0000						1,012		•				
0,000 0.107 0.293 0.537 0.779 0.000 0.141 0.335 0.571 0.800 0.000 0.174 0.379 0.609 0.822 0.000 0.203 0.421 0.645 0.842 0.000 0.228 0.457 0.676 0.860 0.000 0.246 0.484 0.699 0.871 0.000 0.258 0.501 0.713 0.877 0.000 0.262	0	000'0		0.262										
0,000 0.141 0.335 0.571 0.800 0,000 0.174 0.379 0.609 0.822 0,000 0.203 0.421 0.645 0.842 0,000 0.228 0.457 0.676 0.860 0,000 0.246 0.484 0.699 0.871 0,000 0.258 0.501 0.713 0.877	-10	00000		0,293	0.537		0,953							
0,000 0.174 0.379 0.609 0.822 0,000 0.203 0.421 0.645 0.842 0,000 0.228 0.457 0.676 0.860 0,000 0.246 0.484 0.699 0.871 0,000 0.258 0.501 0.713 0.877	-20	00000												
0,000 0.203 0.421 0.645 0.842 0,000 0.228 0.457 0.676 0.860 0,000 0.246 0.484 0.699 0.871 0,000 0.258 0.501 0.713 0.877	-30	00000		0,379										
0,000 0.228 0.457 0.676 0.860 0,000 0.246 0.484 0.699 0.871 0,000 0.258 0.501 0.713 0.877 0,000 0.262	4.0	0000		0.421	. 0.645	0,842								
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Ä	Position Error	Table B4a	in the x Coordinate Caused by the Velocity Error for $\delta = 0^{\circ}$
	on [		

	Pos	ition E	rror in	the x C	?oordina	Table B4a	B4a sed by tl	Table B4a Position Error in the $\times$ Coordinate Caused by the Velocity Error for	ity Err		<i>δ</i> = 0°	
_	R1 = A12*	A12*((A12**2	*2 + S	SING**2)/(A22**2	)/(A224	**2 + A	+ A12**2):**1/2	1**1/2	FOR	œ	DEC =	0
PLI	HA (+0R-) 0	15	30	45	9	12	0,	105	120	135	150	165
0.5	1,000	0.966	0.866	0.707	0.500	0.259	000.0					
80	0,985	0.952	0.856	0.702	0,498	0,259	0.000					
2.0	0,940	0,912	0.827	0.686	0,493	0,258	0.000					
60	0,866	0.846	0.781	0.661	3,484	0.257	000.0					
20	0,766	0.757	0.719	0.630	0,473	0.255	000.0					
0	0,643	0.650	0.648	0.594	0.462	0,254	0.000					
30	0,500	0.529	0.573	0.559	0,451	0,252	000.0					
20	0,342	0.405	0,563	0.528	0.441	0,251	000.0					
10	0.174	0.298	0.452	0.507	0,435	0,250	0.00.0					
0												
-10	0,174	0.298	0.452	0.507	0,435	0,250	000.0					
120	0,342	0.405	0.503	0.528	0,441	0,251	000.0					
30	005'0	0.529	0,573	0.559	0,451	0,252	000.0					
140	0,643	0.650	0.648	0.594	0,462	0,254	000.0					
-50	992'0	0,757	0.719	0.630	0,473	0.255	000.0					
-60	0,866	0.846	0,781	0.661	0,484	0,257	000.0					
-70	0,940	0,912	0.827	0,686	0,493	0.258	000.0					
-80	986,0	0.952	0.856	0.702	0,498	0.259	000.0					
06-	1,000	0,966	0,866	0.707	0.500	0,259	000.0					

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Table B4b	Caused
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	Position

	Po	sition E	rror in	the y C	cordina (	Table B4b Position Error in the $_{\it Y}$ Coordinate Caused by the Velocity Error for	4b ed by t	he Velo	city Err		<i>δ</i> = 0°		
R2	10	SING * ( A12 * +2	+	SING**2)**1/2	(**1/5	F 0.R		DEC =	0				
HAC	HA (+0R-) 0	15	30	45	0.9	/2	06	105	120	135	150	165	180
<b>.</b>			٠										
0.6	000.0	0.259	0.500	0.707	0,866	0,966 1	1,000						
0 8	00010	0.255	0.494	0.702	0,863	0,965 1	1.000						
7.0	000.0	0,244	0.478	0,686	0,853	0,962 1	1,000						
60	00000	0,227	0.451	0.661	0,839	846.0	1.000						
50	00000	0,203	0,415	0.630	0,820	0,952 1	1.000						
04	00000	0.174	0.374	0.594	0.800	0.947	1.000						
30	00000	0.142	0.331	0.559	0,781	0,941 1	1.000						
20	00000	0.109	0,291	0.528	0.764	0,937 1	1.000						
10	00000	0.080	0.261	0.507	0,754	0,934 1	1.000						
0	000.0	0.067	0.250	0.500	0,750	0.933 1	1.000						
-10	00000	0.080	0.261	0.507	0,754	0,934 1	1.000						
-20	000.0	0.109	0.291	0.528	0,764	0,937	1.000						
02-	000.0	0.142	0.331	0.559	0,781	0,941 1	1,000						
140	00000	0.174	0.374	0.594	0.800	0.947 1	1,000						
.50	00000	0.203	0.415	0.630	0,820	0,952	1,000						
09-	00000	0.227	0,451	0.661	0,839	856.0	1.000						
-70	000 0	0.244	0.478	0.686	0,853	0,962 1	1,000						
-60	000.0	0,255	0.494	0.702	0,863	965	1,000						
05-	00000	0.259	0.500	0.707	0,866	996'0	1.000						

	δ = 9°
	Error for
	Velocity
le B5a	aused by the
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	sition

		180																			1.000
	6	165																			0,966 1,000
δ = 9°	DEC =	150																		984	0.866
Table B5a Position Error in the $\times$ Coordinate Caused by the Velocity Error for	F 0R	135																		6 0,729	0,707
ity E		120																		0,526	0.500
е Veloc	+ SING**2)/(A22**2 + A12**2))**1/2	105																ii, 345	315	. 286	9,259
5a d by tl	12**2	06										1,012	0.676	0,403	0.267	0.187	0,132	0.091	0,058	0,028	000.0
Table B5a	*2 + A	75					0.380	0,424	0.484	0.580	0,757	6/6'0	0,537	0.174	0,016	0.0.0	0.125	0.167	0,201	0.231	0,259
7 ordinate	/(A22*	9			0.548	0,567	0,586		0,632	0,677	0,765	0.880	0,363	0,065	0,220	0,301	0,355	0,399	0,437	0,470	0.500
he x Co	NG**2)	4.			0.740	0.740	0,731	0,716 0,605	869.0	0.687	969.0	0,725	0.185	0,256	0.386	0.462	0.525	0,582	0.632	0,674	0.707 0,500
or in tl		30				0,858	0.817	0,761	969.0	0.625	0.564	0.524	0.032	0.340	0.444	0.533	0.621	0.703	0,773	0.829	0.866
ion Err	A12*((A12**2	15		0.980	0.966 0.881	0,924	0.857	0.767 0.761	0.658	0,536	0.411	0.303	0.066	0,284	0,405	0.534	0.658	0.767	0,858	0.925	996.0
Posit	R1 = A12*(	HA(+0R-) 0		1,012	0,994	0,945	0,868	0,764	0,637	0,491	0,330	0,158	0,018	0,193	0,363	0,521	0,664	0,787	0,886	0,957	1,000
		PFI	0 6	80	20	60	<b>80</b>	0	30	20	10	0	-10	-20	-30	0 7	-50	9	-70	90	06-

Table B5b

	Pos	Table Bbb Position Error in the $v$ Coordinate Caused by the Velocity Error for $\delta$	rror in	the v C	oordina	Table B5b te Caused	Bob sed by t	he Velo	city Er	ror for	δ = 9°		
	R2 # SING	SING + (A12++2	*2 + S	SING**2)**1/2	)**1/2	i.	F 0 78	DEC =	. 0				
<b>3</b>	HA(+0R-) 0	15	3.0	45	9	75	06	105	120	135	150	165	180
4													
0.5													
<b>a</b>	000 0	0,000 0.262											
70	000.0	0.258	0.501	0.713	0,877								
9	00000	0,246	0.484	0.699	0,871								
90 0	000.0	0.228	0.457	0.676	0,860	0.977							
4	000 0	0,203	0,421	0,645	0,842	0,974							
30	00000	0.174	0.379	0.609	0,822	0,968							
20	00000	0.141	0.335	0.571	008.0	0,961							
10	000'0	0.107	0.293	0.537	0,779	0,953							
0	00000	0.079	0,262	0,512	0,762	0.945	1.012						
-10	000'0	0.067	0.250	0.501	0,752	956,0	1,012						
-20	000'0	0.082	0.261	0.504	0,750	0,935	1,011						
0 10	000 0	0.112	0,290	0.523	0,756	0,933	1.009						
4.0	000.0	0.146	0,331	0,553	0,770	0,934	1.007						
100 NO	000.0	0.178	0.376	0.589	0,789	986.0	1.005						
09-	000 0	0.207	0,418	0.626	0,810	0,943	1,003	9.49					
-70	00000	0,231	0,455	0,660	0,832	0,951	1,001	976.					
-80	000.0	0,248	0.483	0,688	0,851	0.959	1,000	6.972	0,875	0,716	0.506		
06-	00000	0,259	0.500	0.707	0,866	0,866 0,966	1,000	996.	0,866	0,707	0.500	0.500 0,259 0.000	0.000

	Pos	ition E	rror in	Table B6a Position Error in the $\times$ Coordinate Caused by the Velocity Error for	ordinat	Table B6a e Caused b	B6a ed by th	e Veloc	ity Err		$\delta = 18^{\circ}$		
R3.		A12*((A12**2	+	SING**2)/(A22**2	)/(A22		+ A12**2))**1/2	) *** 1/2		FOR	DEC =	-18	
HA (+0R-) PH I		0	15 30	0 7.	09	75	06	105	120	135	150	165	180
0.6													
90													
7.0	1,051	1.022	8										
09	1,028	3 1.007	7 0.940	0 0.823									
50	0,975	5 0,963	3 0,921	1 0.836	0,698								
0.4	0,892	2 0.891	1 0,881	1 0,838	0,742								
30	0,781	962.0 1	6 0.823	3 0.830	0,788	0,680							
50	0,647	0.680	0 0,752	2 0.818	0,839	0,799							
10	0,494	0.550	0 0,675	0.804	0,897	0,938							
0	0,325	0.415	5 0.596	5 0.778	0,925	1,019	1.051						
-10	0,146	0,288	8 0.470	0,623	0,754	0.857	0,923						
-20	0,037	0.071	1 0.027	0.180	0,362	0.543	969.0						
30	0,219	0.276	6 0.271	0.142	0,063	0.294	0.509						
04-	0,394	0.414	4 0.406	5 0.304	0,112	0.128	0,372	. 585					
0.00	0,557	0,553	3 0,517	0.409	0,224	0,014	0,269	i, <b>5</b> 05					
• • 0	0,704	0.685	5 0.621	0.496	0,308	0,071	0,187	£,436	0,654				
-70	0,829	0.801	1 0.717	7 0.575	0,378	0.140	0,118	. 374	909'0	0,797	0.937		
.60	0,928	968.0	6 0.800	0.645	0,441	0.201	0.057	316	0,555	0,758	0.912	1,009 1.041	1.041
06-	1,000	996'0 (	998.0 9	0.707	0,500	0.259	0.000 0.259	6.259	0,500	0,707	998.0	996'0	1.000

	$\delta = 18^{\circ}$	
	or for 8	
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Table B6b	Caused	
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	Posi	

	Д	)OSi	tion Er	nor in	the v	oordina	Table B6b	B6b	Table B6b Position Error in the " Coordinate Caused by the Velocity Fruor for s = 18°	oity Ev	non for	1 4	0	
R2	S	ÖZ	SING+(A12++2	2 + S	SING**2)**1/2	**1/2		FOR	DEC =	-18				
HA (+8R+)	-8-	0	15	30	2	9	15	00	105		¥	ic c	4	9
PH.			i											
0.5														
0.0					r									
7.0	00000	00	0.272											
0 9	00000	00	0.267	0.520 0.742	0.742									
000	000,0	00	0,255	0,255 0,503	0.729	0,908								
<b>4</b> 0	00000		0,235	0,474	0.705	868'0								
30	000 0	00	0,209	0.209 0.436 0.672	0.672	0,880	1.014							
20	00000		0.178	0.178 0.391	0.632	0,856	1,008							
10	00000		0.143	0.343	0.590	0,829	0.997							
0	0.0	00	0,108	0,000 0,108 0,298	0.550	0,801	0,984	1,051						
-10	000.0	00	0.078	0.264	0.519	0,777	0.970	1.050						
-20	000'0		0,067	0,067 0,250	0,502	0,759	956.0	1.046						
-30	00000		0,085	0.261	0,503	0,750	0.944	1.039						
0 4 1	00000		0.117	0.294	0.521	0,753	0,936		1,031 1,016					
0	00000		0,153	0.338	0.553 0,765	0,765	0,933	1,022	1,013					
09-	00000		0.187		0.386 0.593 0,786	0,786	0,935	1,013	1.005	0.910				
-70	00000		0.217	0.431	0,431 0,635 0,812	0,812	0,942	1,006	0,994		0,903 0,742	0.526		
- 60	000.0		0.241		0.674	0.840	0,470 0,674 0,840 0,953		1,002 0,980 0,888 0,730	0,888	0,730	0.519	0.519 0,269 0,000	0.000
06-	00000		0.259		0.707	0,866	996.0	1,000	0,500 0,707 0,866 0,966 1,000 0,966 0,866	0,866	0.707	0.500	0.259 0.000	0.000

	Posit	Table B7a Position Error in the $\times Coordinate$ Caused by the Velocity Error for	or in th	ne × Co	ordinat	Table B7a te Caused	7a ed by th	e Veloc	ity Err	or for	$\delta = 27^{\circ}$		
F 1.		A12*((A12**2	2 + SIN	(6**2)	+ SING**2)/(A22**2	*2 + A	+ A12**2))**1/2	)**1/2	FOR	œ	DEC =	-27	
HA(+8R-)	•	15	30	<b>4</b>	9	75	06	105	120	135	150	165	180
0.6													
0 8													
70													
60	1,121 1.099	1.099											
30	1,094	1,080	1.035	0.951									
0	1,033	1,030	1.014	0,970 0,883	0,683								
30	0,941	0,951	996.0	0,972	926'0								
20	0,821	0.845	0.901	0.957	0,985	0,968							
10	0,675	0.716	0.815	0.926	0,926 1,015	1,061							
0	0,510	0.571	0.714	0,872	1,005	1,092	1.122						
-10	0,328	0.419	0,593	0.763	0,903	1,003	1,057						
-20	0,137	0,256	0,256 0,389 0,522 0,662	0.522	0,662	0,795	0.902						
92	0,059	0.080	0.016 0.172	0.172	0,358	0,551	0,723	958.0					
0+-	0,252	0.281	0.242	660.0	0,099 0,107	0,336	0,557	0,744					
-50	0,439	0.439	0.397	0.272	0,072	0.168	0,414	0,637	0,819				
09-	0,611	0,611 0,594 0,530	0.530	0.400	0,204	0,400 0,204 0,035	0,291	0,536	0,747 0,914 1,031	0,914	1,031		
-70	0,765	0.738	0,738 0,654 0,511 0,312 0,073 0,185 0,440	0,511	0,312	0.073	0.185	0,440	0.670	0,860	0,860 1,000	1,085	1.11
-80	968.0	0.864	0,864 0,768	0,613	0,409	0.169	060.0	0,409 0,169 0,090 0,348	0,587		0,790 0,944	1,041	1.073
05-	1,000	996.0	0,966 0,866	0.707	0.500	0,259	0.00.0	0,500 0,259 0,000 0,259	0,500	0.707	0,500 0,707 0,866 0,966 1,000	996.0	1.000

	Po	sition	Err	or in	the $y \in C$	oordina	Table B7b ate Caused	B7b sed by t	Table B7b Position Error in the $\gamma$ Coordinate Caused by the Velocity Error for	city Er	ror for	: 8 = 27°	o	
R2	91	SING * (A12 * * 2	2**2		+ SING**2)**1/2	**1/2	. لغد	FOR	DEC =	-27				
HA (+0R-) Phi	9R-2		15	30	<b>4</b>	09	15	06	105	120	135	150	165	180
0.5														
60														
7.0														
09	00000	0.290	0.0											
000	000 0	0.284		0,555	0.792									
0	000000	0.270		0,535	0.779	0,971								
30	000'0	0.248		0.504	0,752	0,960								
20	000'0	0.220		0.461	0.714	0,938	1,082							
10	000,0	0.186		0,411	0.667	0,907	1,072							
0	000 0	0.148		0.357	0,616	0,870	1,055	1,122						
-10	000'0	0.100		0.306	0.567	0,832	1,032	1,119						
-20	000'0	770.0 (		0.266	0.527 0,796	962,0	1,006	1.109						
-30	00000	990.00		0.250	0.504	0,768	0.980	1,093	1,084					
3	000'0	060.00		0,263	0.502	0,752	0,958	1,073	.077					
150	000.0	0.126		0.301	0.522	0,752	0,941	1,052	1,062	0,970				
-60	000'0	0,165		0.352	0.560	0,766	0,933	1,032	. 041	0,958	0,791	0.561		
-70	00000	0.201		0.406	0.608	0,792	0,935	1,015	1.016	0,935	0,776	0.554	0,288	000.0
0 8 9	000.0	0.233	33 0,	0,457	0.659	0,828		0,947 1,004	066	0,903	0,747	0.533	0,277	000.0
061	00000	0,259		0.500	0,707 0,866	998.0		0,966 1,000	996'0	998.0	0,707	0.500	0,259 0	000.0



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A LUNAR RADAR NAVIGATION CONCEPT			
A final report on one phase of the problem; wo:	rk is continuir	ıg.	
A. Shapiro, E.A. Uliana, and B.S. Yaplee			
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c.	9b. OTHER REPORT	NO(S) (Any o	ther numbers that may be assigned
d.	this report)		
In addition to security requirements which appl transmittal outside the agencies of the U.S. Gov Director, Naval Research Laboratory, Washing	ernment mus	t have pri 90.	or approval of the
A navigation system concept is described to radar system. By measuring the range and range the location of the receiver on the earth's surfactude. Lunar radar observations have shown the curacies equivalent to ±30 m can be achieved. The measurement accuracy could be improved by a urements where rapid readout is required, the factor of 5 to 10. Placing three transmitters at will provide worldwide coverage. A transmitter mitting aperture of 170 m would supply a S/N retion with a dipole antenna receiver. One possible both a search and track mode is given.  A mathematical analysis of the coverage at (a) worldwide coverage is available, (b) time coage, but the time distribution of the coverage of the effective location accuracy is a function of the coverage.	age rate of the ace can be detat lunar range By placing a tactor of 5. It is accuracies appropriate for with an averatio sufficient ole radar recent the effective overage is resarries over a new detate of the effective aries over a new detate ove	receiver ermined i and range ransponde dowever, would prolocations rage powe for relia iver confidericted to nonthly pe	relative to the moon, in both longitude and lati- e rate measurement ac- er on the moon, the basic for operational meas- obably be degraded by a on the earth's surface r of 2 MW and a trans- ble position determina- iguration incorporating accuracy indicates that 50 percent on the aver- eriod, (c) at low latitudes
the effective location accuracy is a function of curacy is obtained at high latitudes.	the moon's de	elination,	and (d) the optimum ac-

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	wr	ROLE	WT	ROLE	WΤ
Bistatic radar						
Lunar navigation concept					}	
Lunar radar						
}		j				

While restricted to some extent in coverage as described above, and more sensitive to systematic errors in the ship's velocity, the lunar radar navigation system, when compared with other worldwide radio navigation systems such as Omega and Transit, can achieve higher accuracies, is less vulnerable to jamming, and can also provide an independent, one-way communication channel. With improved technological developments and active reflectors on the moon, position accuracies of the order of several meters appear possible.